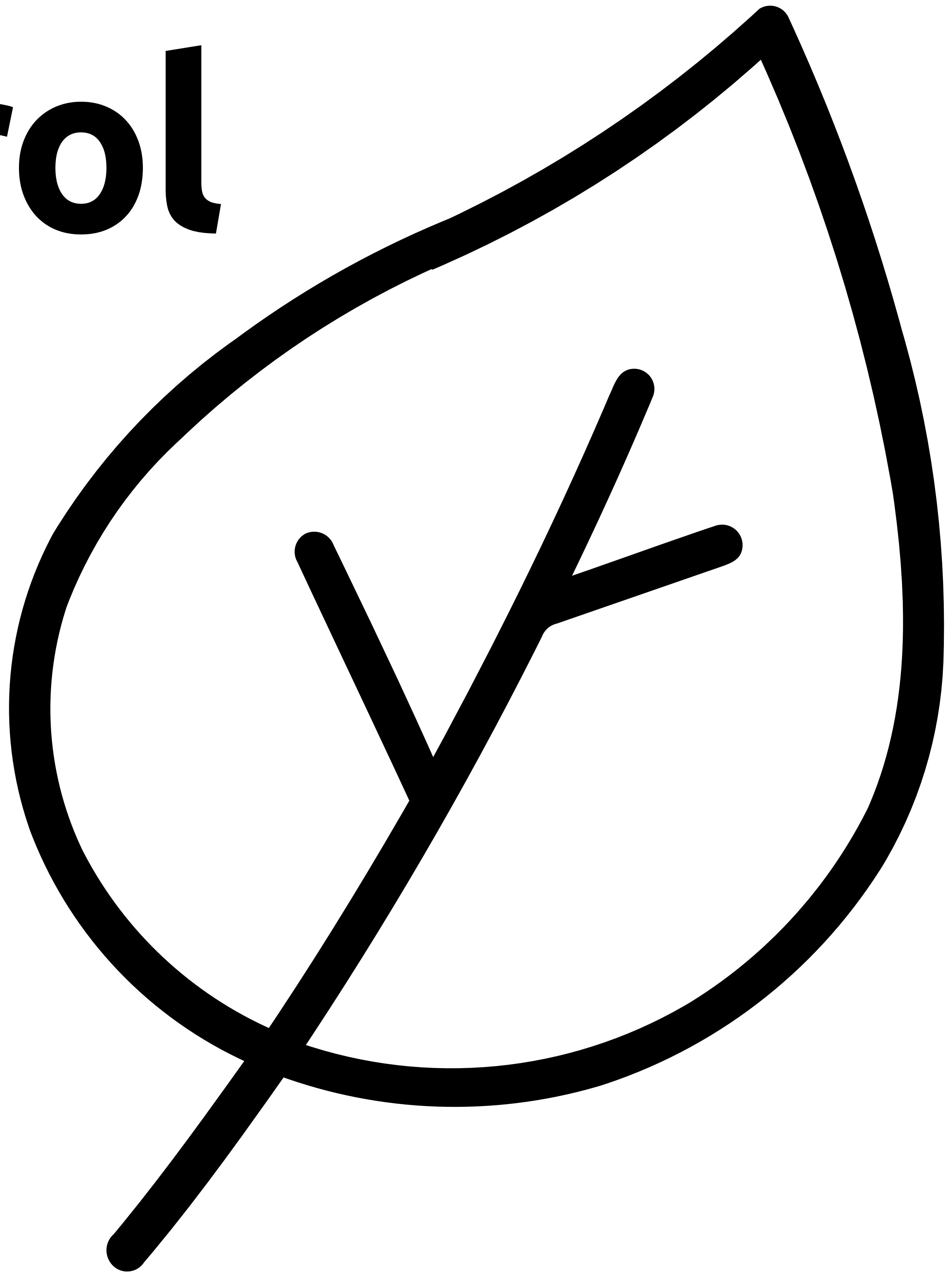
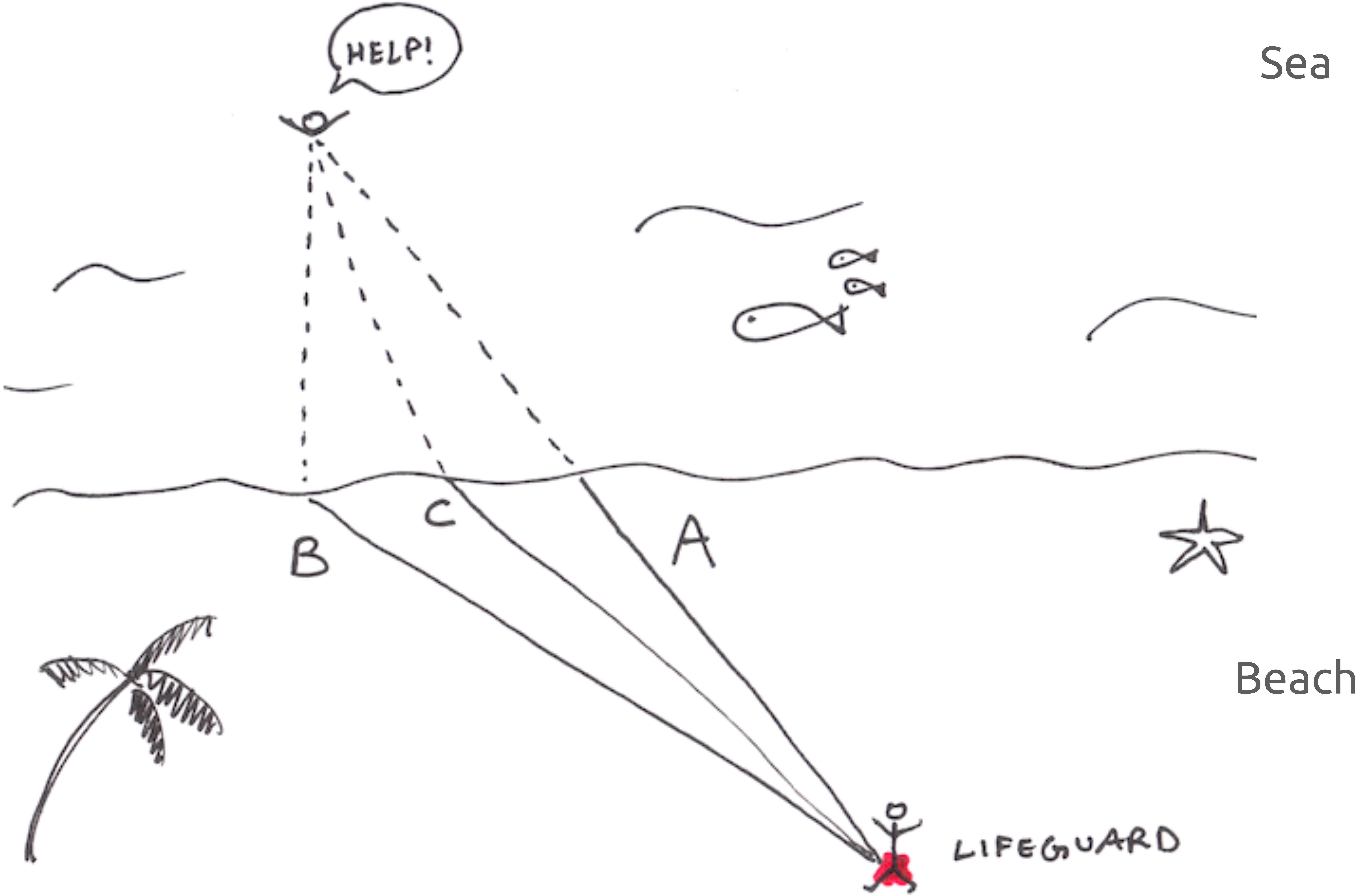
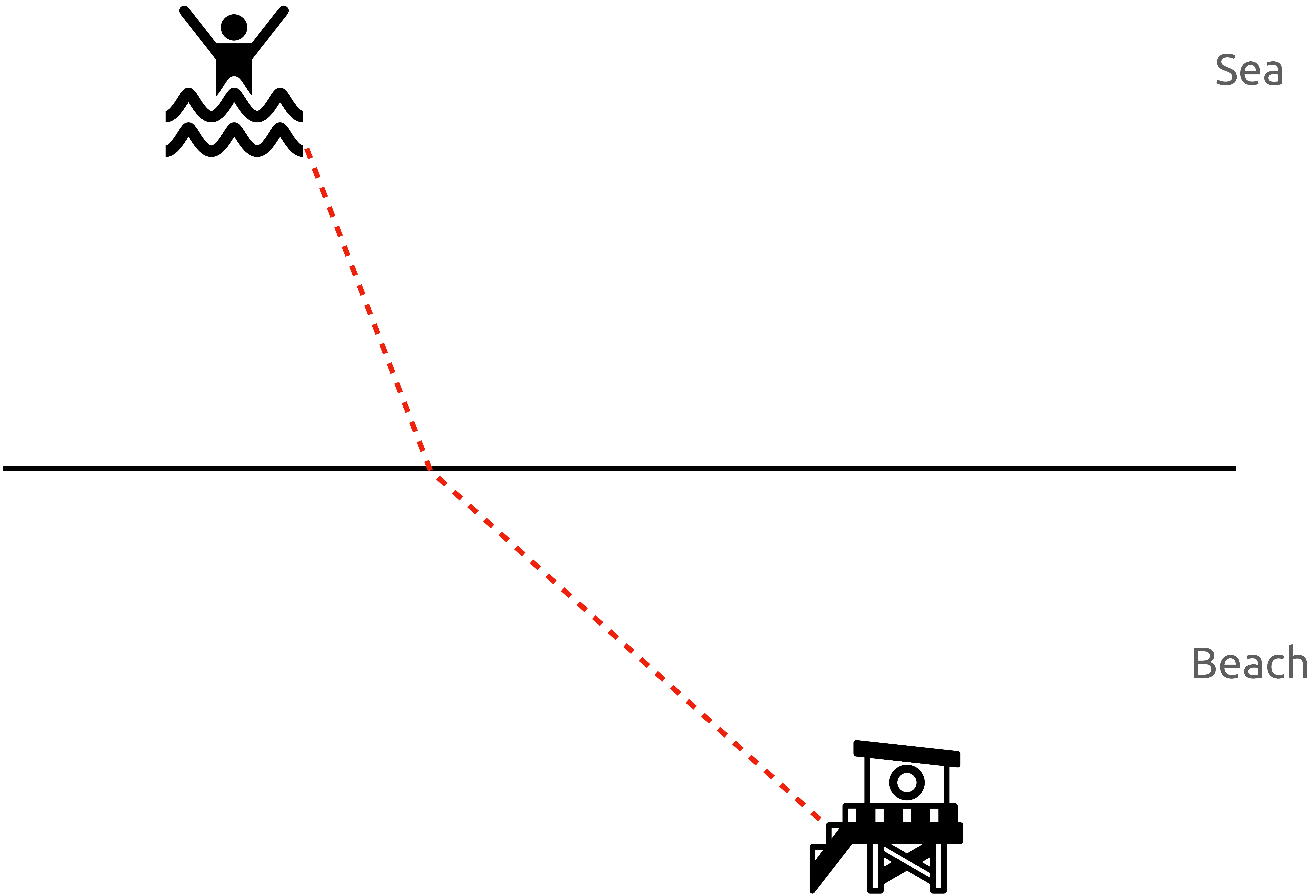
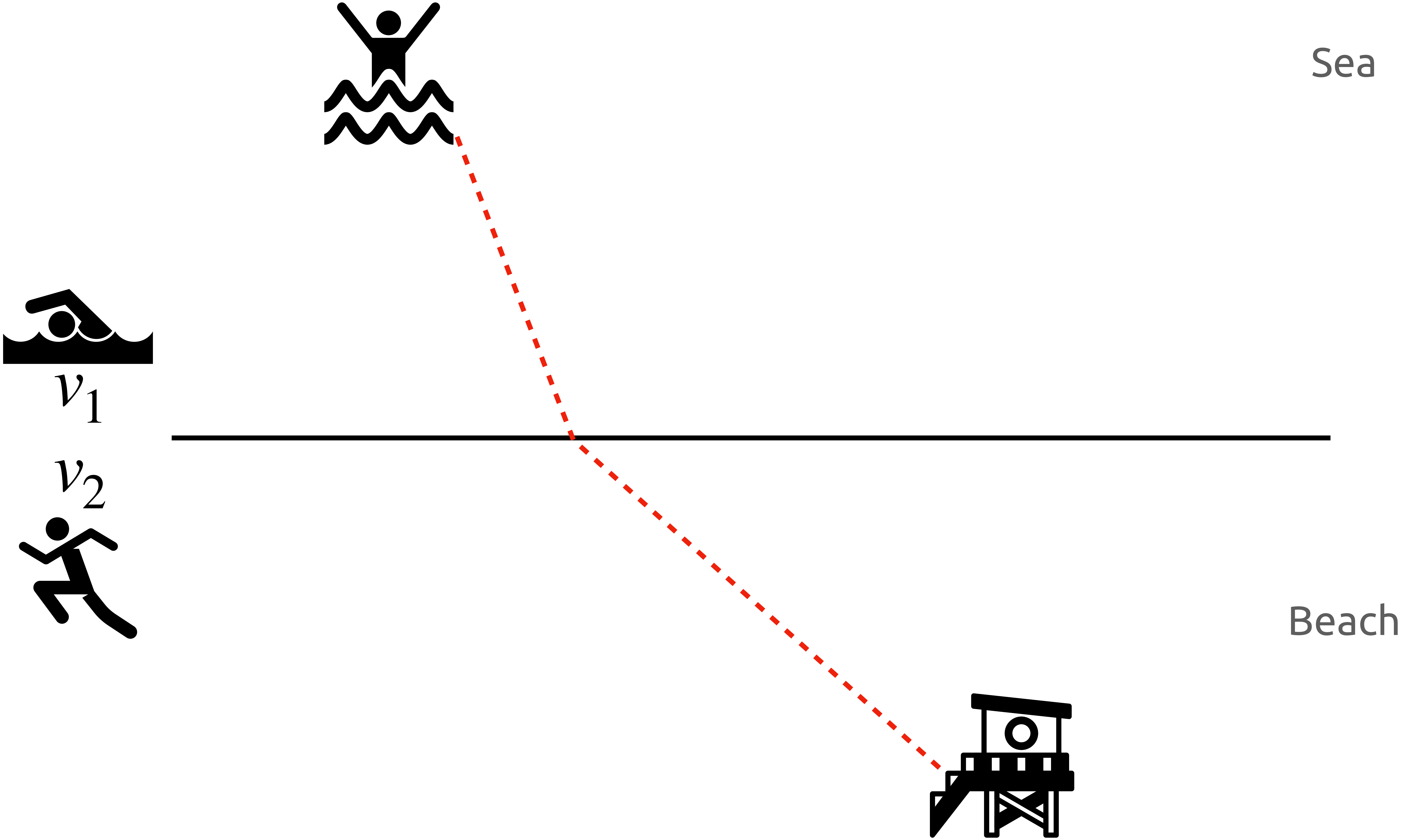
Optimal stomatal control

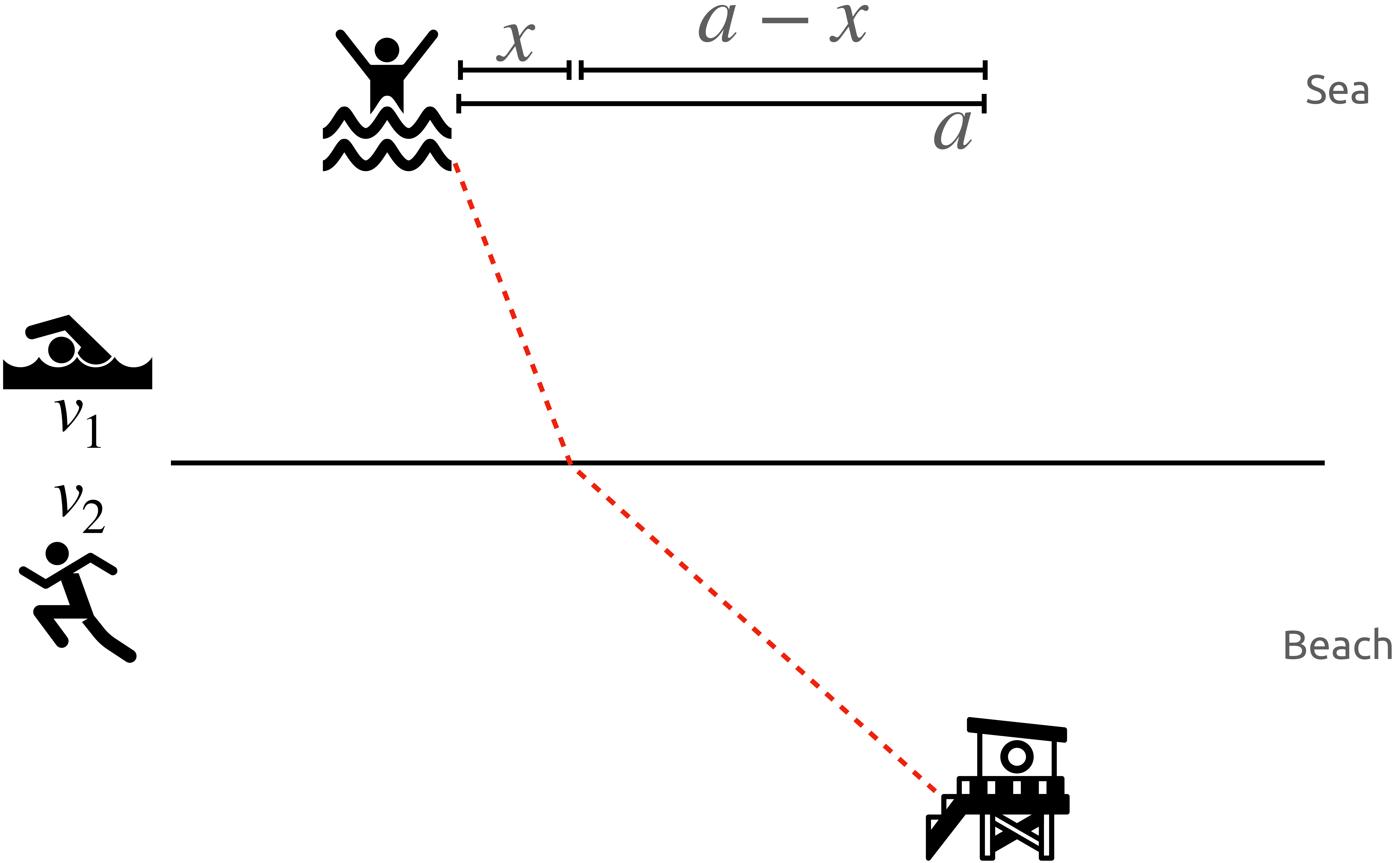


Yair Mau

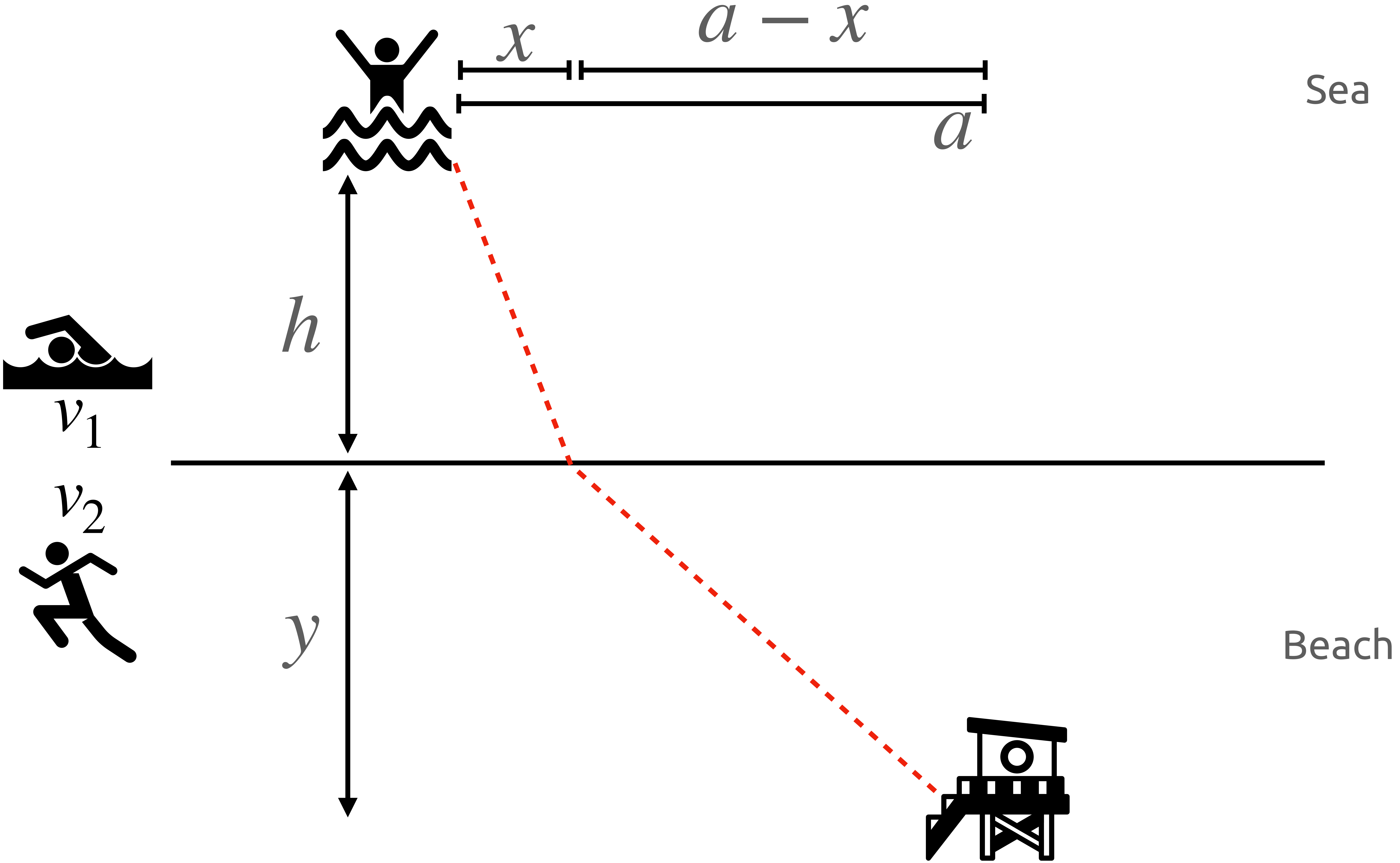

HELP!
Sea
B
C
A
Beach
LIFEGUARD


Sea
Beach

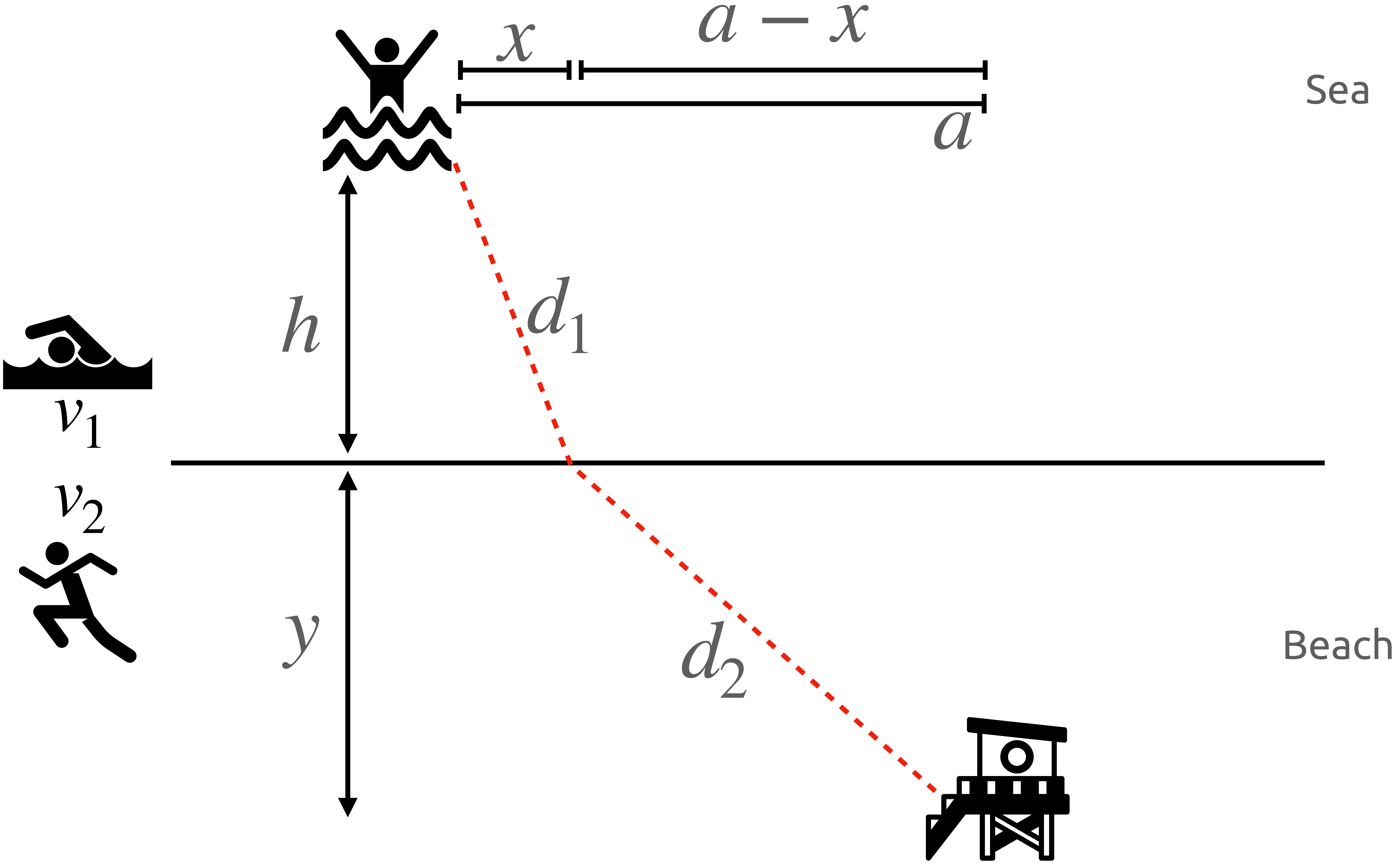

Sea
v_1
v_2
Beach



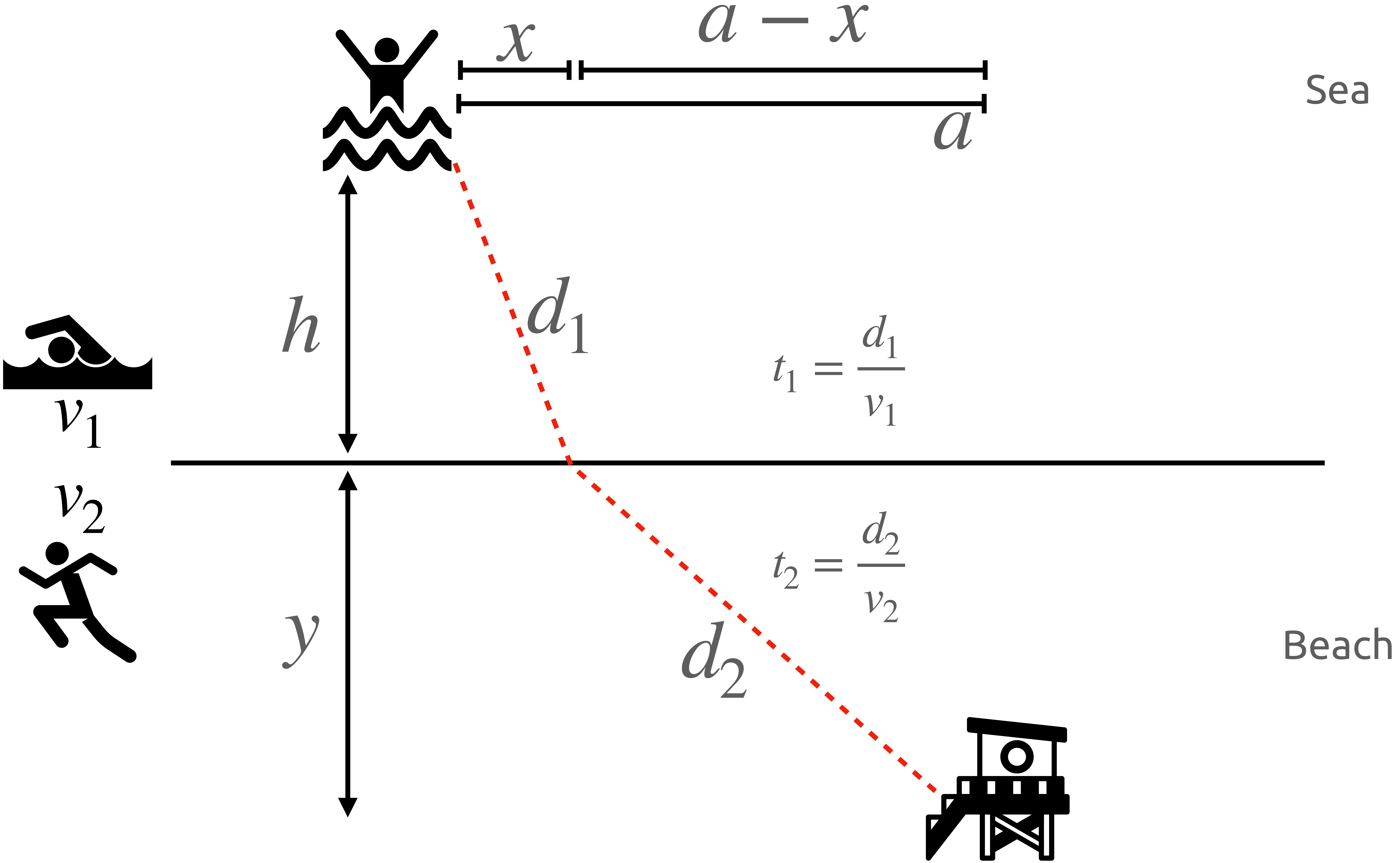
x
$a - x$
a
Sea
v_1
v_2
Beach



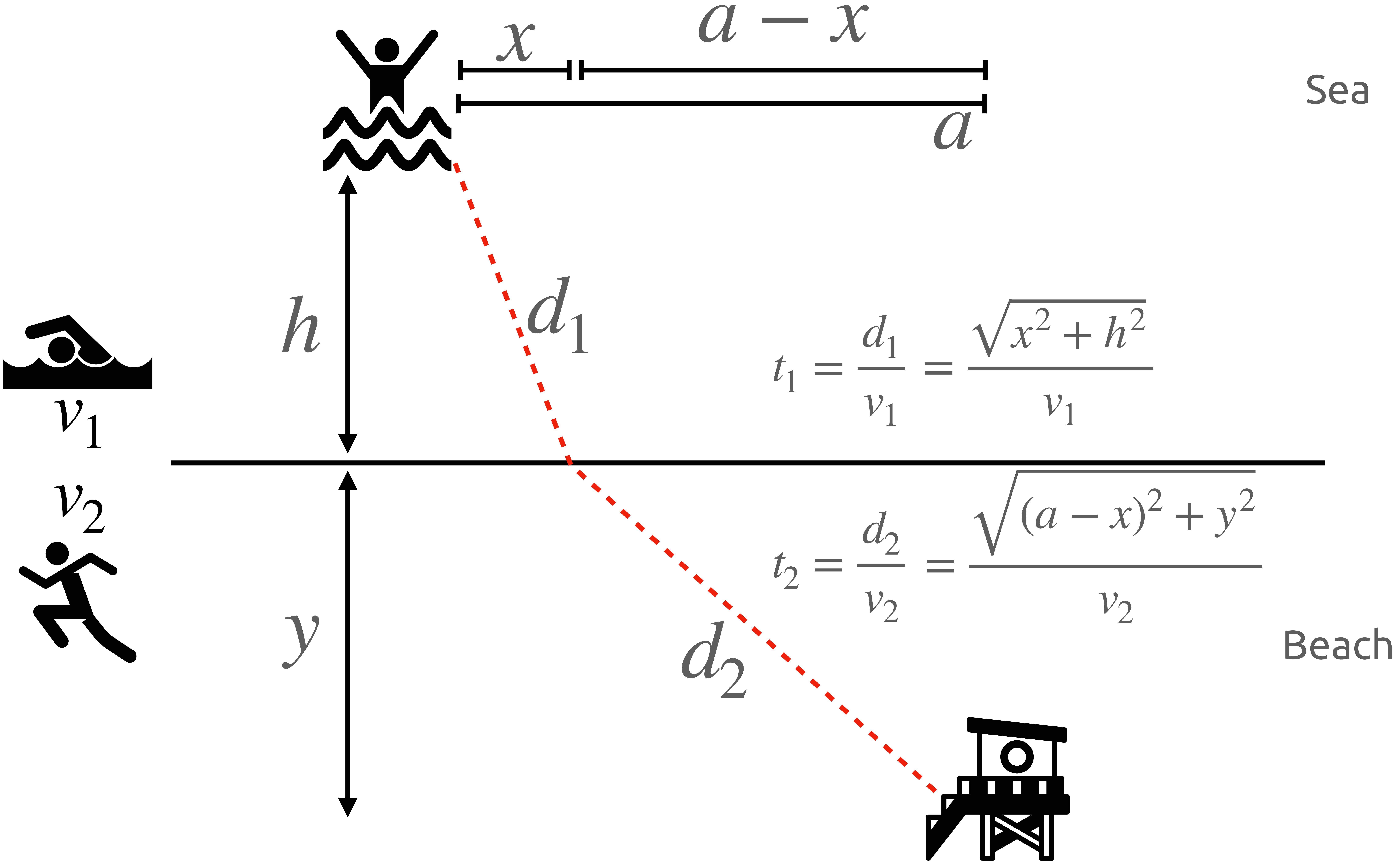
x
$a - x$
a
Sea
h
v_1
v_2
y
Beach



x
a − x
a
Sea
h
d1
v1
v2
y
d2
Beach

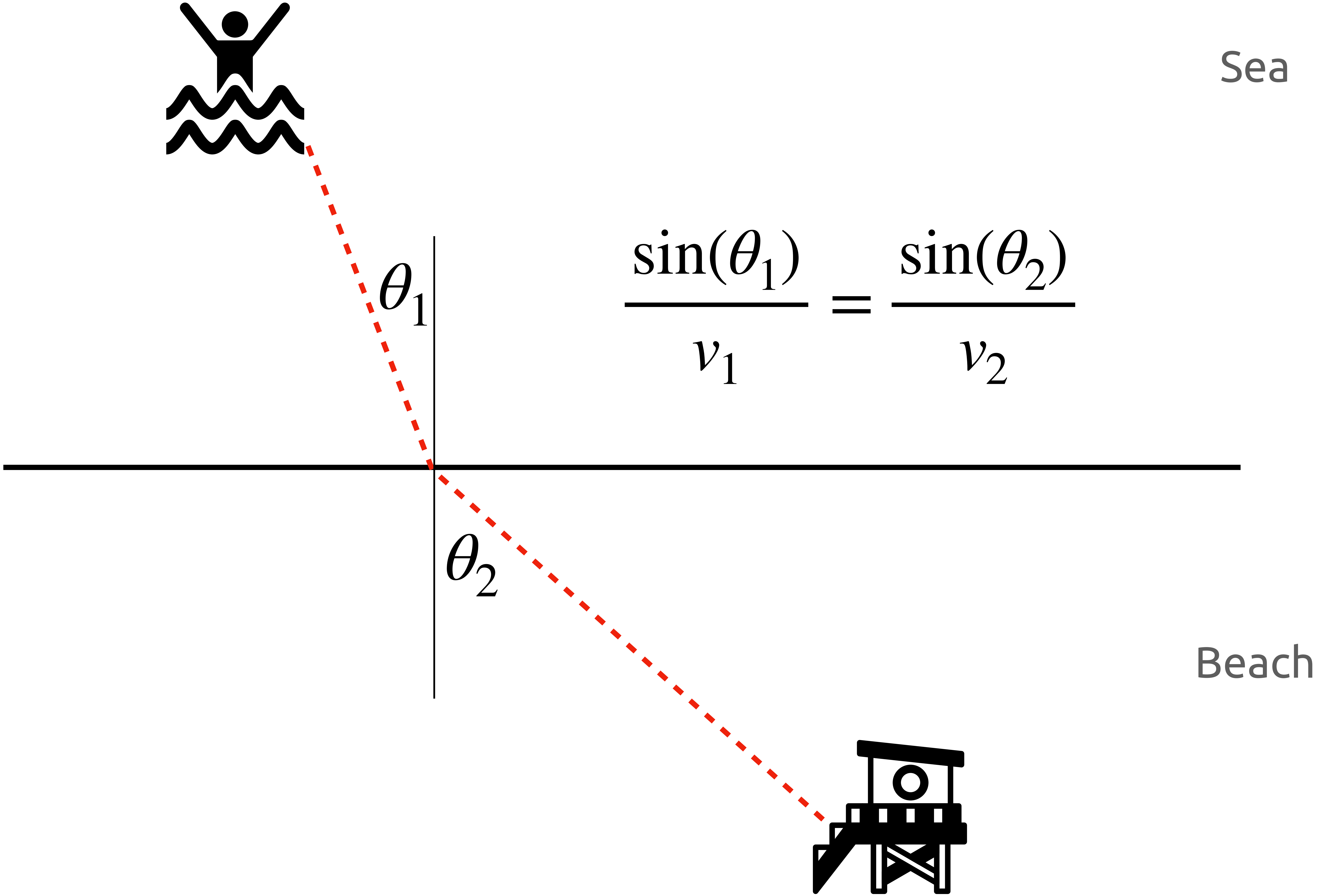


x
$a - x$
a
Sea
h
d_1
v_1
$t_1 = \frac{d_1}{v_1}$
v_2
y
d_2
$t_2 = \frac{d_2}{v_2}$
Beach

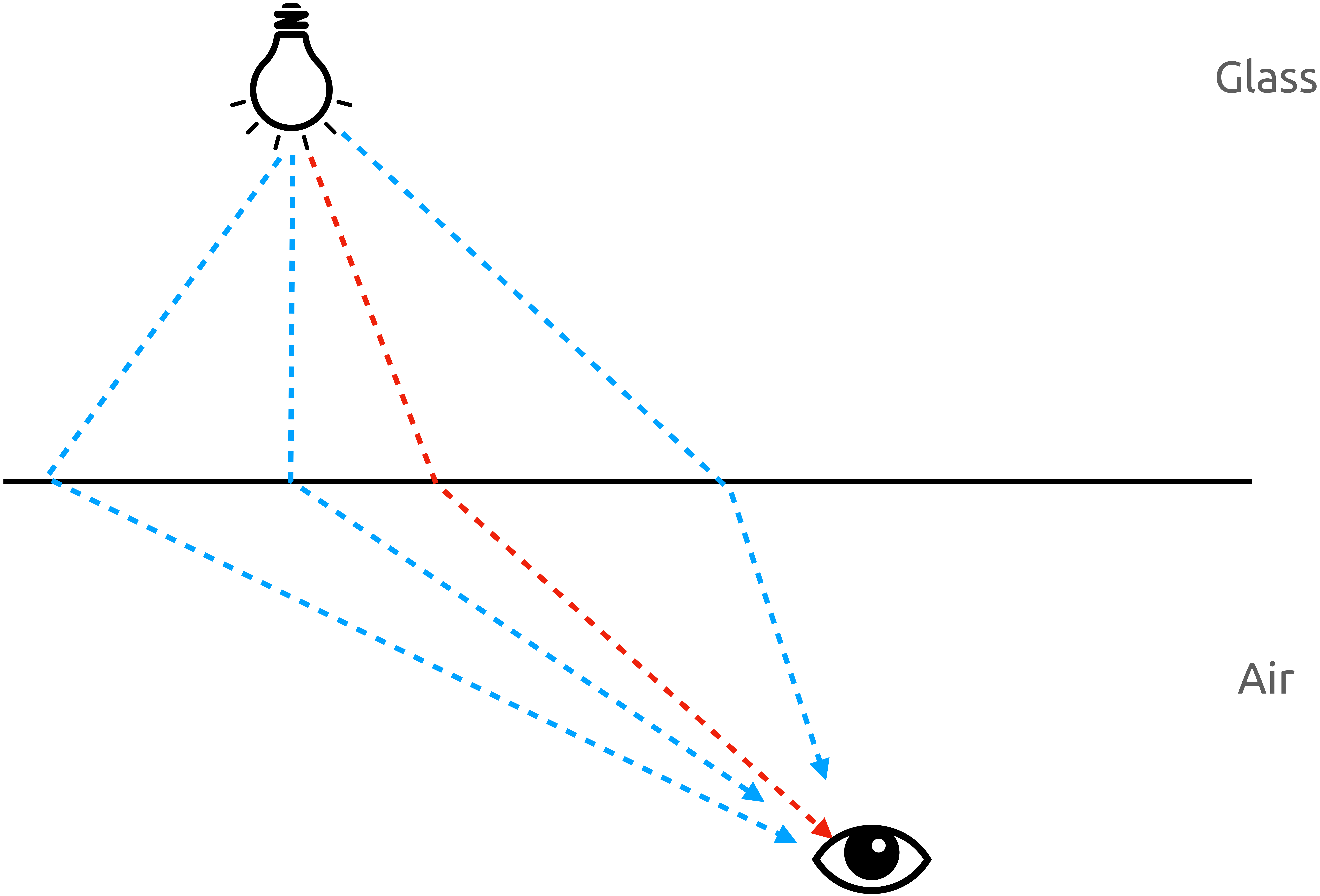


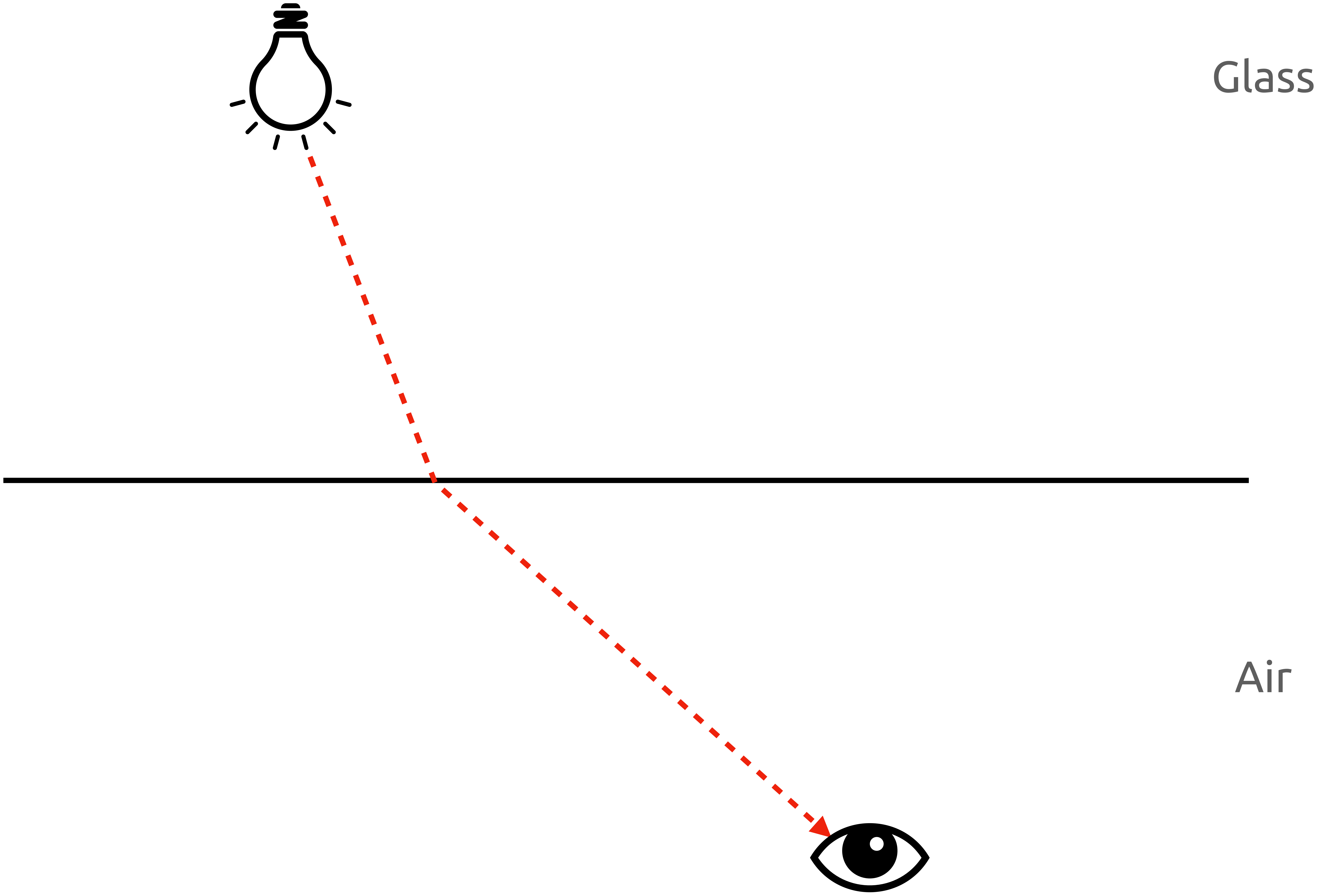
x
$a - x$
a
Sea
h
d_1
v_1
$t_1 = \frac{d_1}{v_1} = \frac{\sqrt{x^2 + h^2}}{v_1}$
v_2
y
d_2
$t_2 = \frac{d_2}{v_2} = \frac{\sqrt{(a-x)^2 + y^2}}{v_2}$
Beach

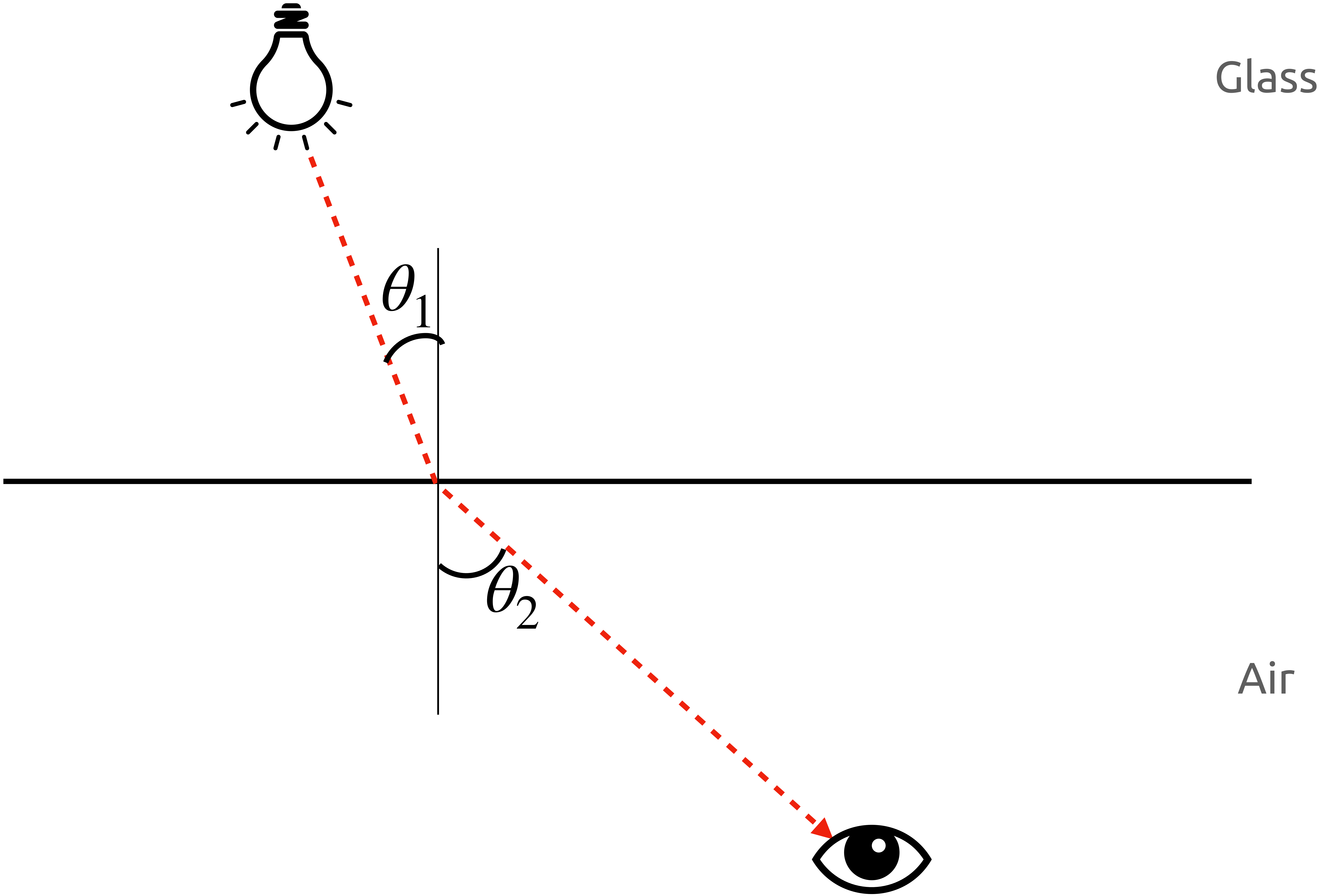
$t_1 + t_2$
Sea
x
Beach



$$\frac{\sin(\theta_1)}{v_1} = \frac{\sin(\theta_2)}{v_2}$$


Glass
Air

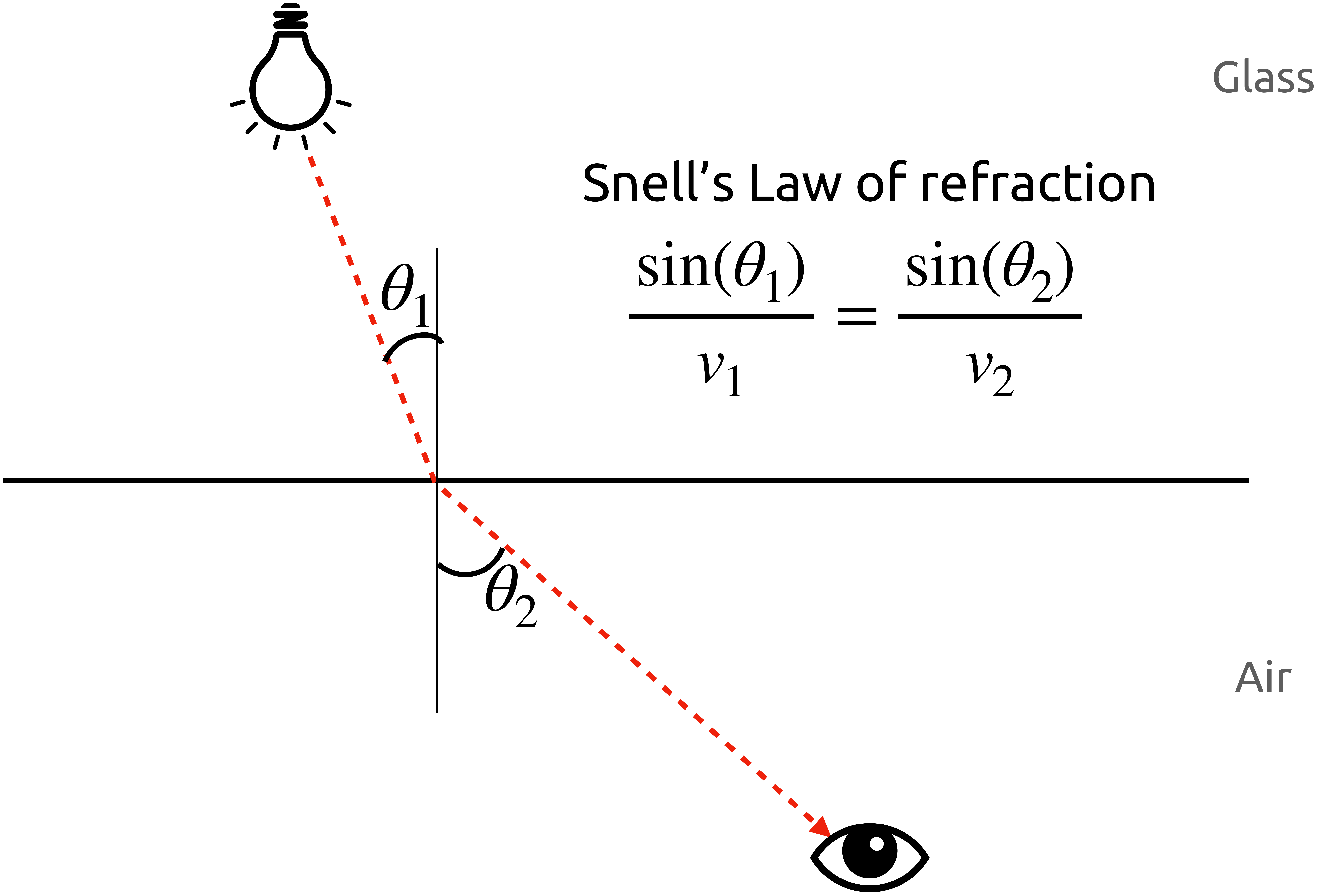

Glass
Air



Glass
θ1
θ2
Air

Snell's Law of refraction

$$\frac{\sin(\theta_1)}{v_1} = \frac{\sin(\theta_2)}{v_2}$$



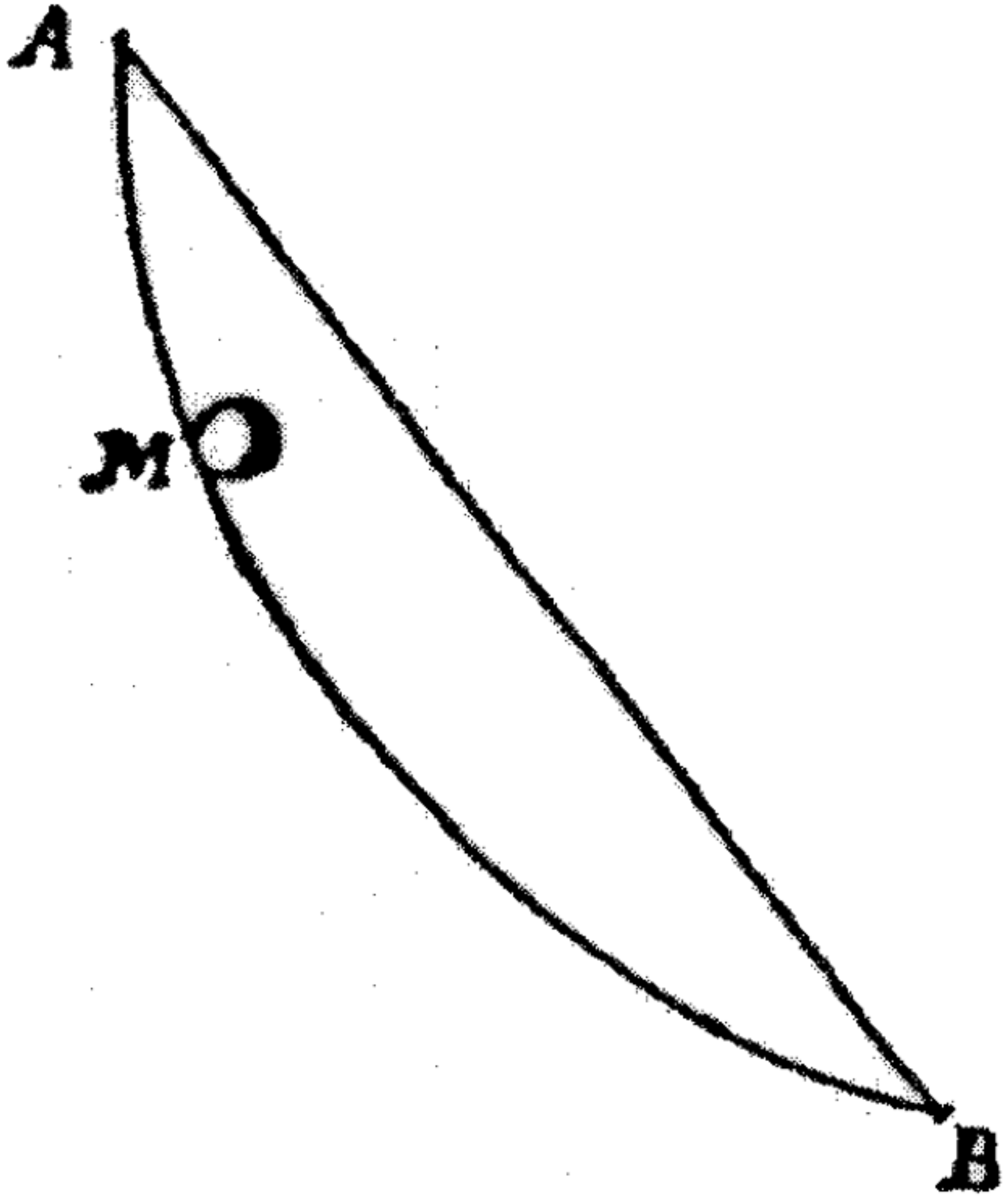
Rough
Felt

Smooth
Felt

Johann Bernoulli

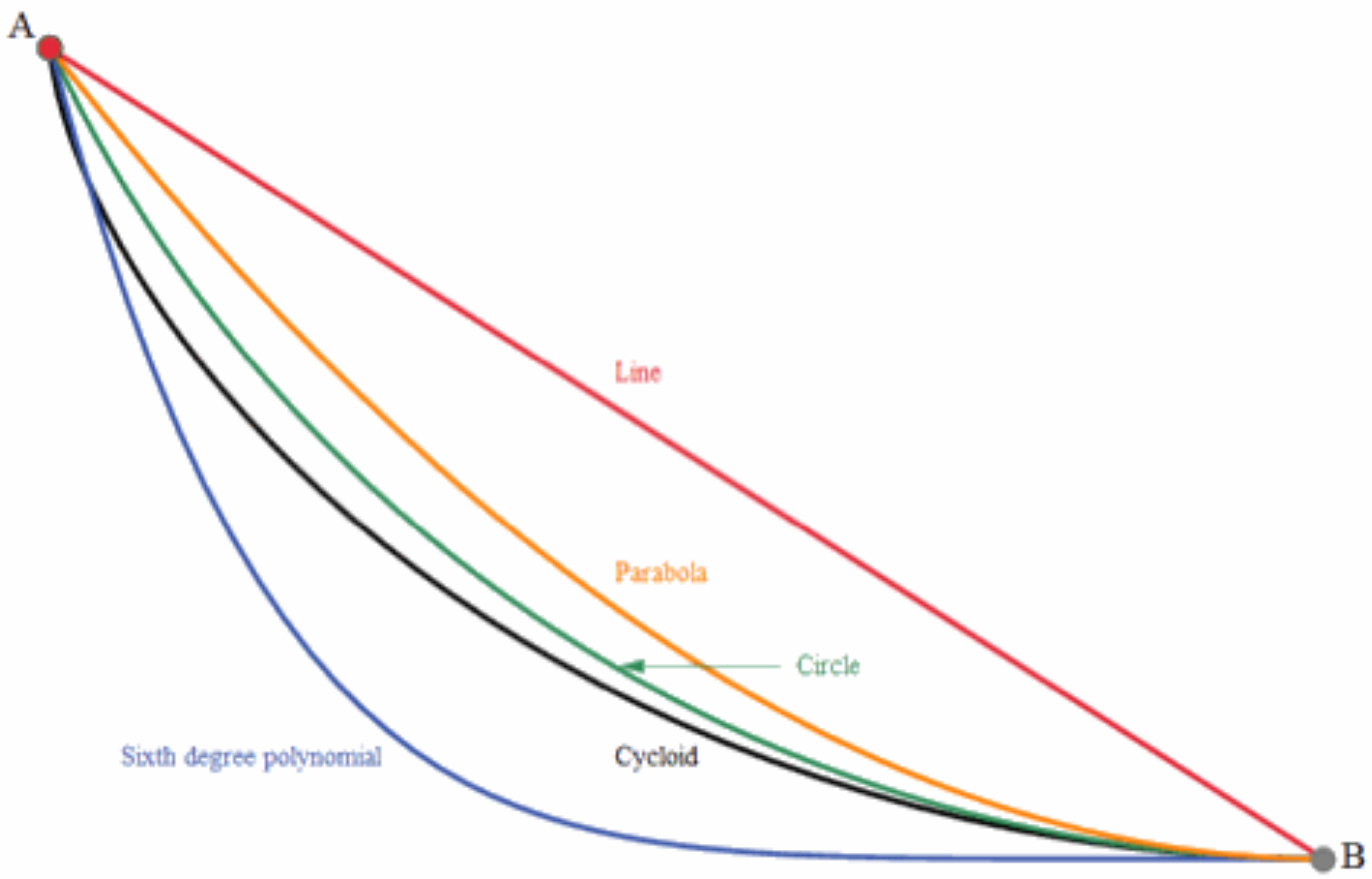
1696

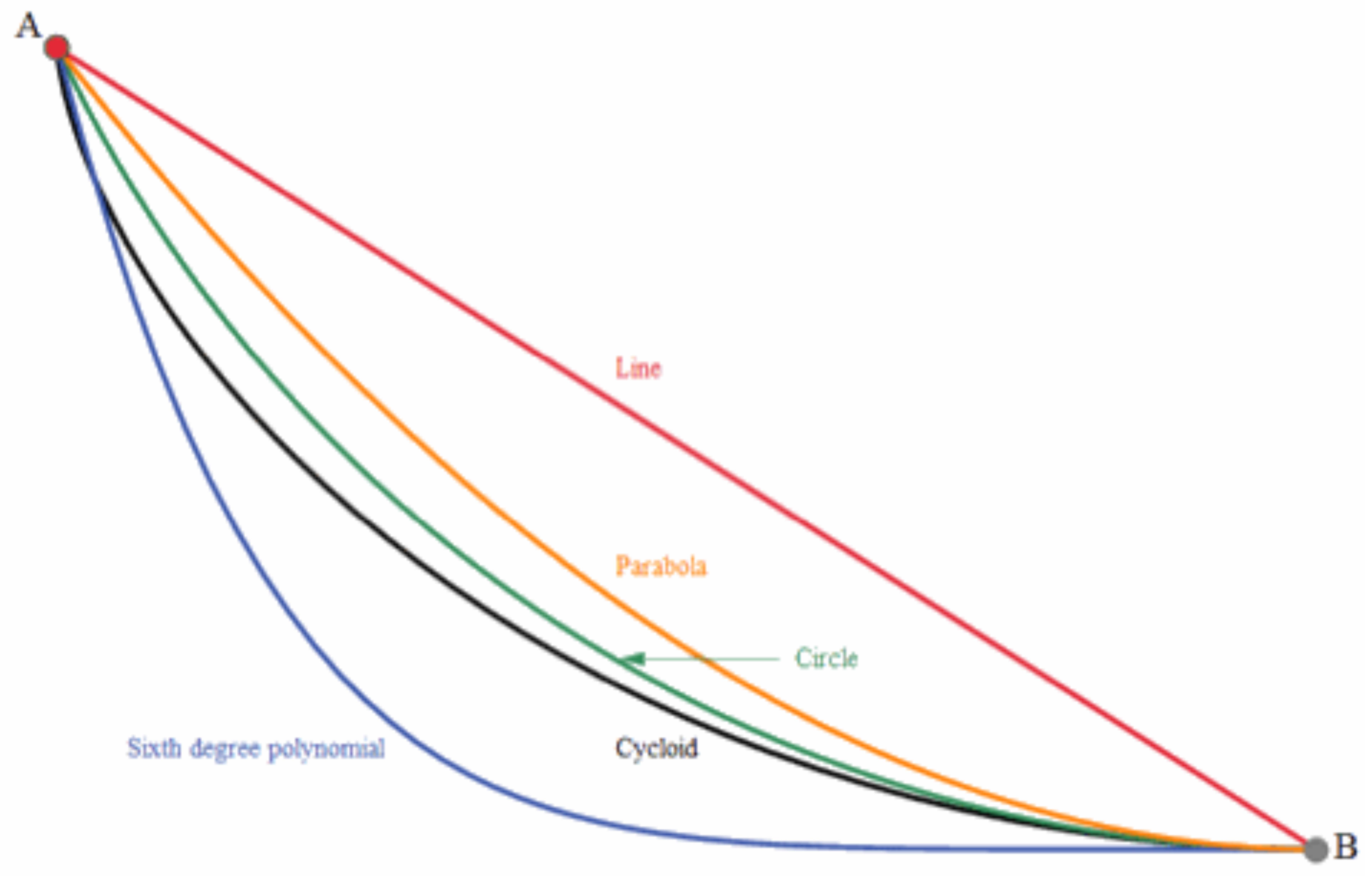
the brachistochrone problem



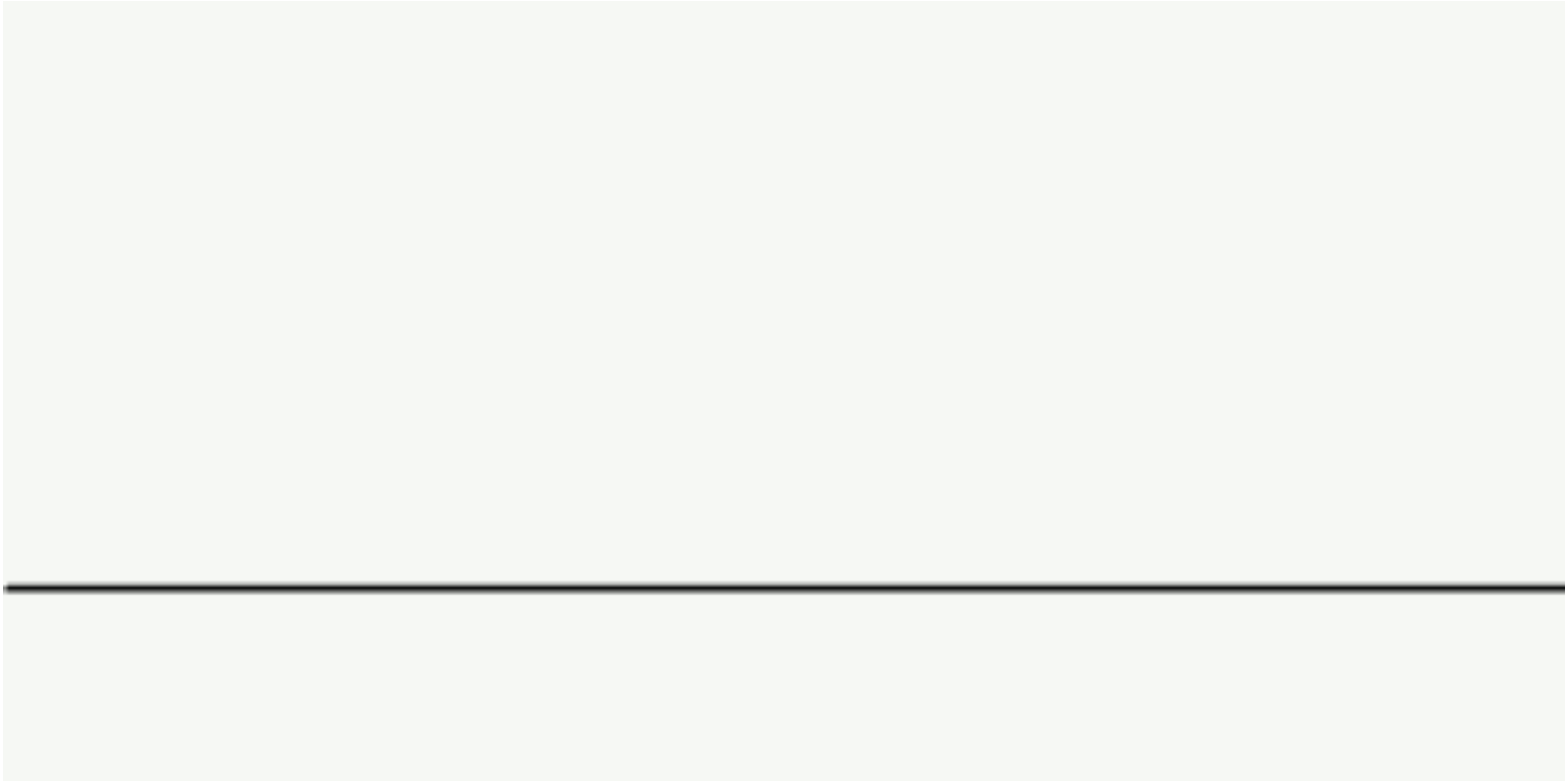
$$x = r(t - \sin t)$$

$$y = r(1 - \cos t)$$

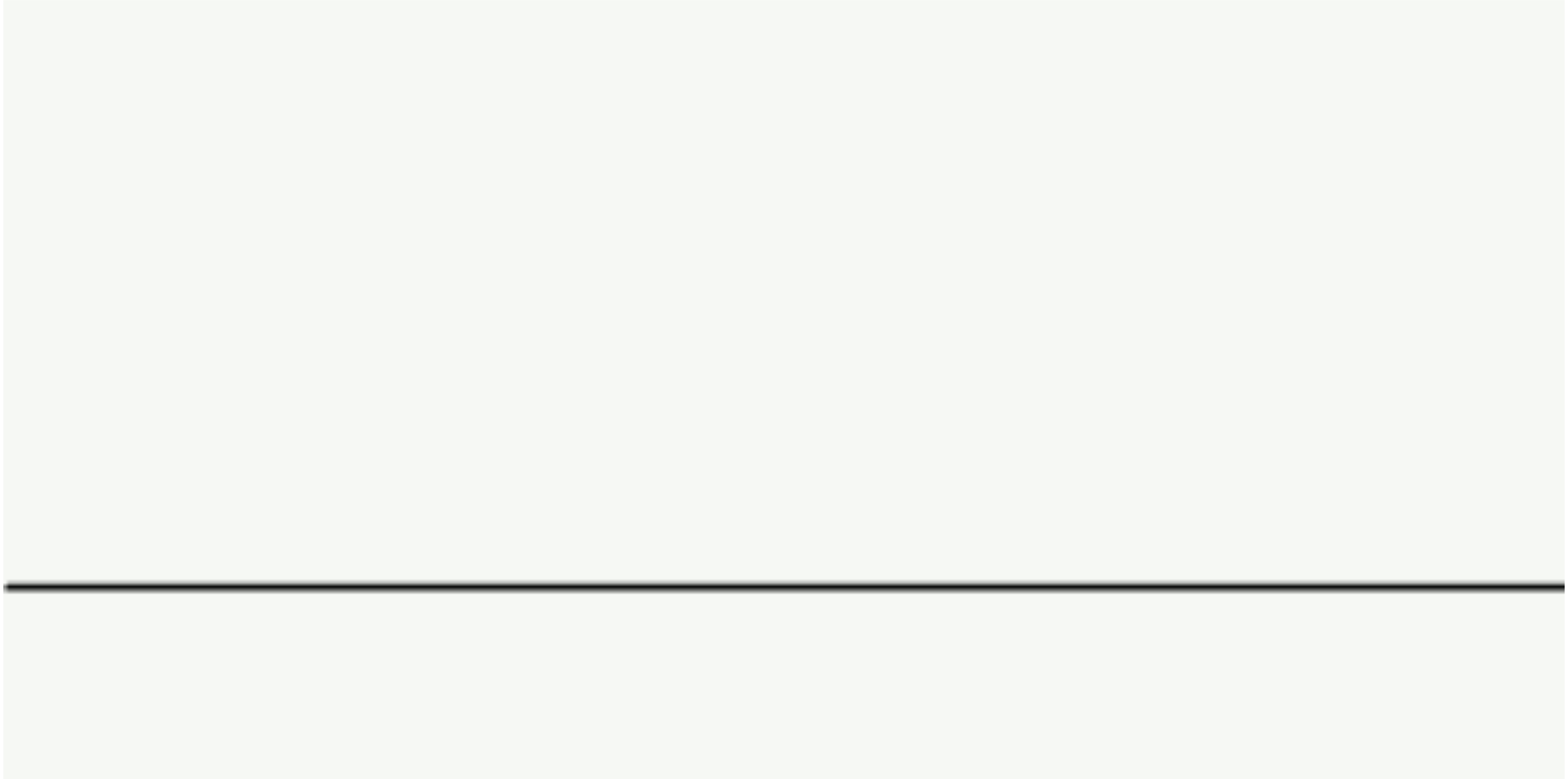

A
Line
Parabola
Circle
Sixth degree polynomial
Cycloid
B


A
Line
Parabola
Circle
Sixth degree polynomial
Cycloid
B

cycloid



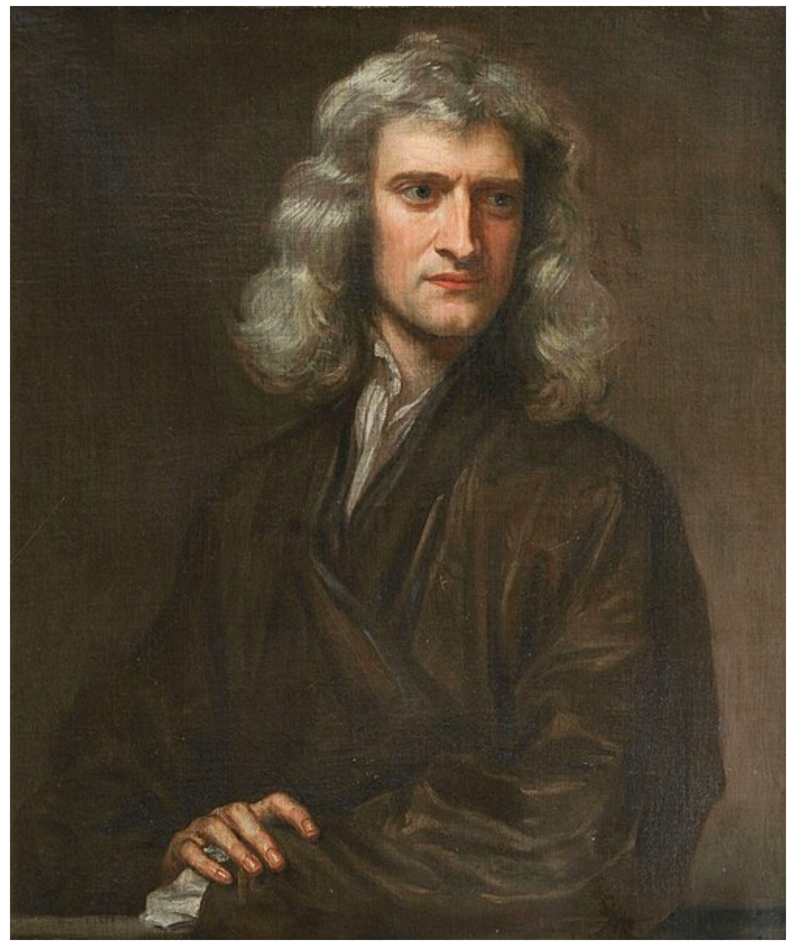
cycloid



Johann Bernoulli

Gottfried Wilhelm Leibniz

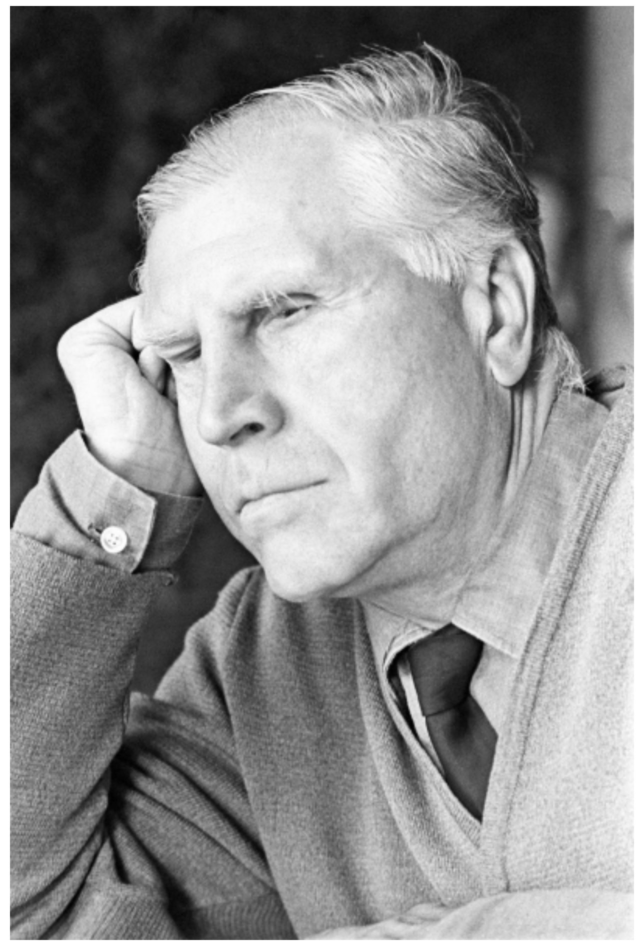
Isaac Newton



Leonhard Euler

Joseph-Louis Lagrange

William Rowan Hamilton



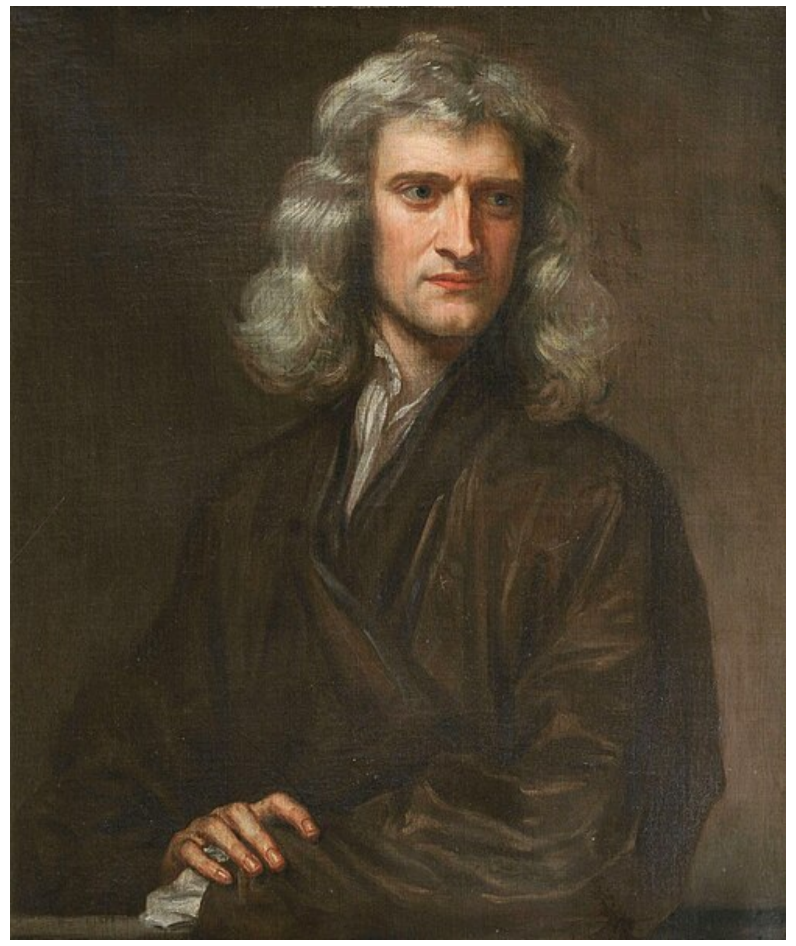
Lev Pontryagin

Richard E. Bellman

Johann Bernoulli

Gottfried Wilhelm Leibniz

Isaac Newton



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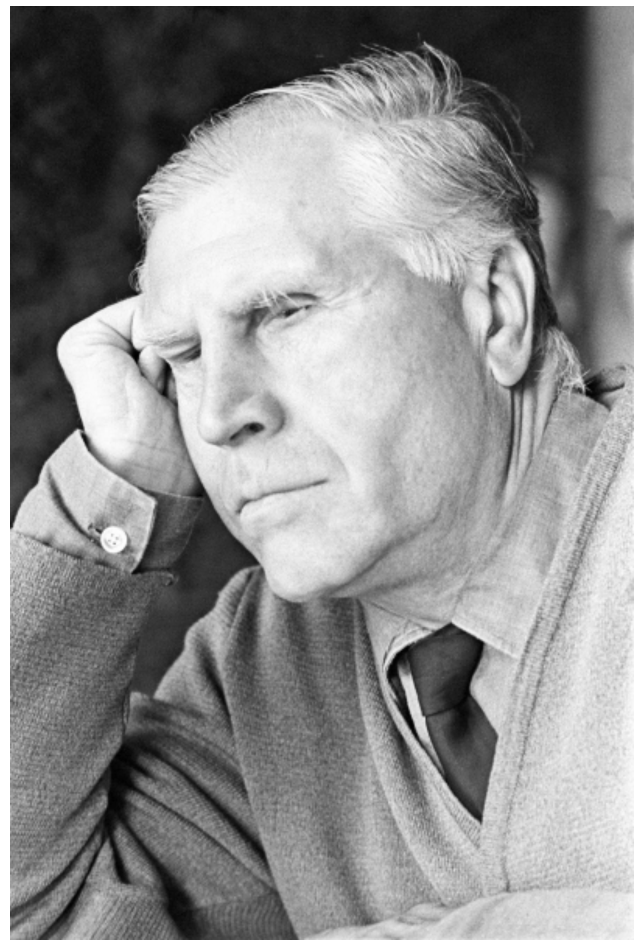


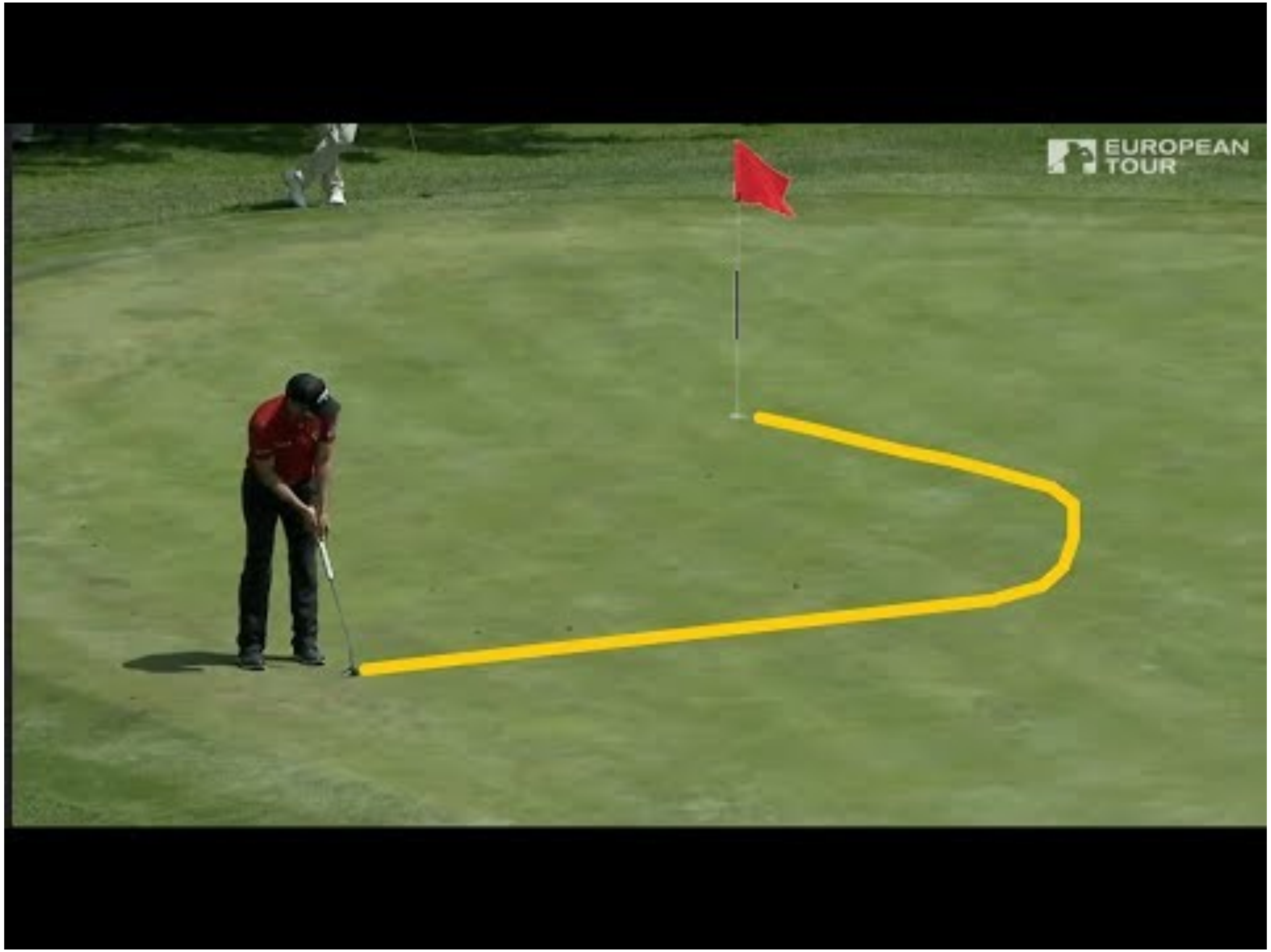
Lev Pontryagin

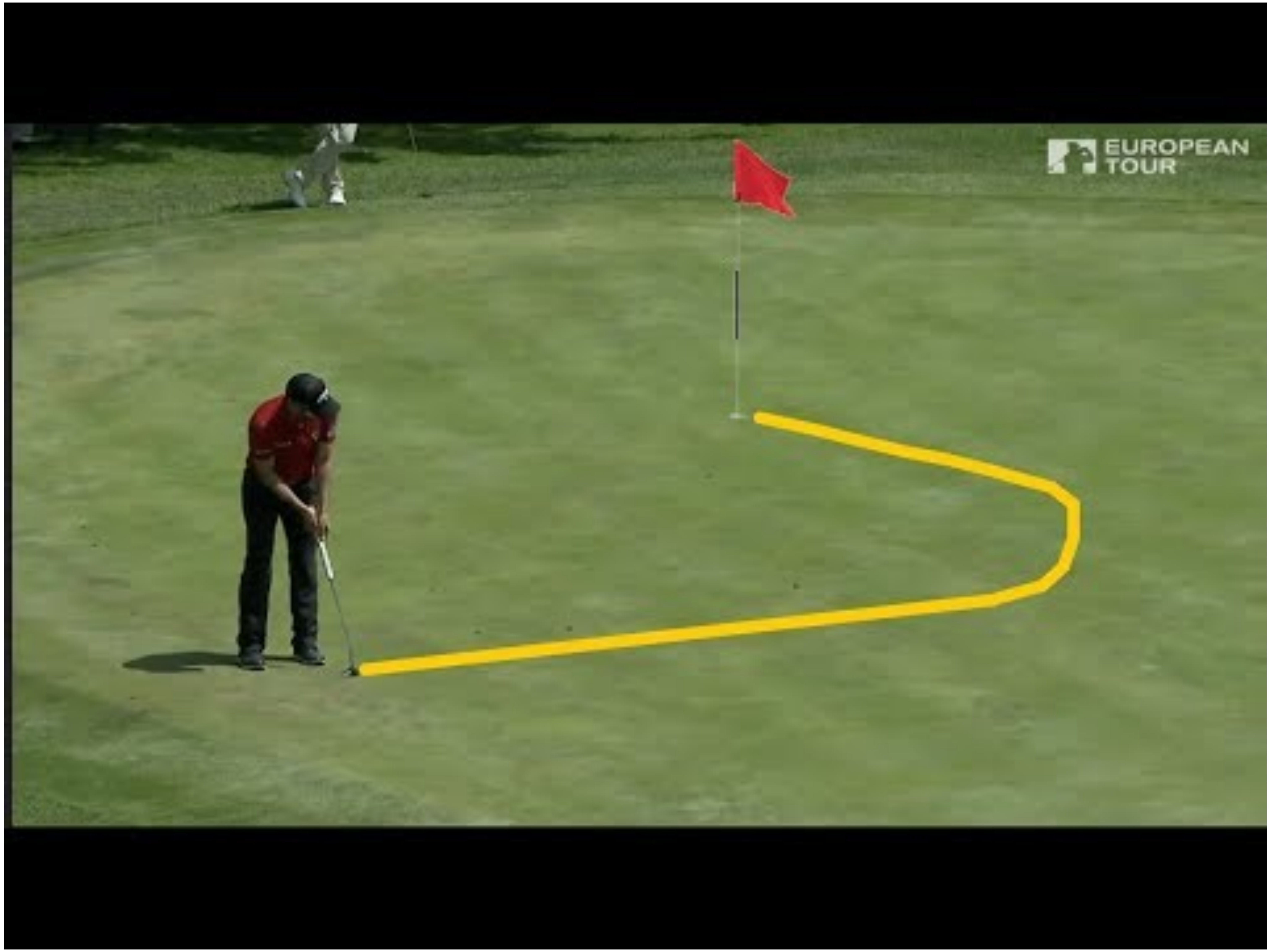
Richard E. Bellman

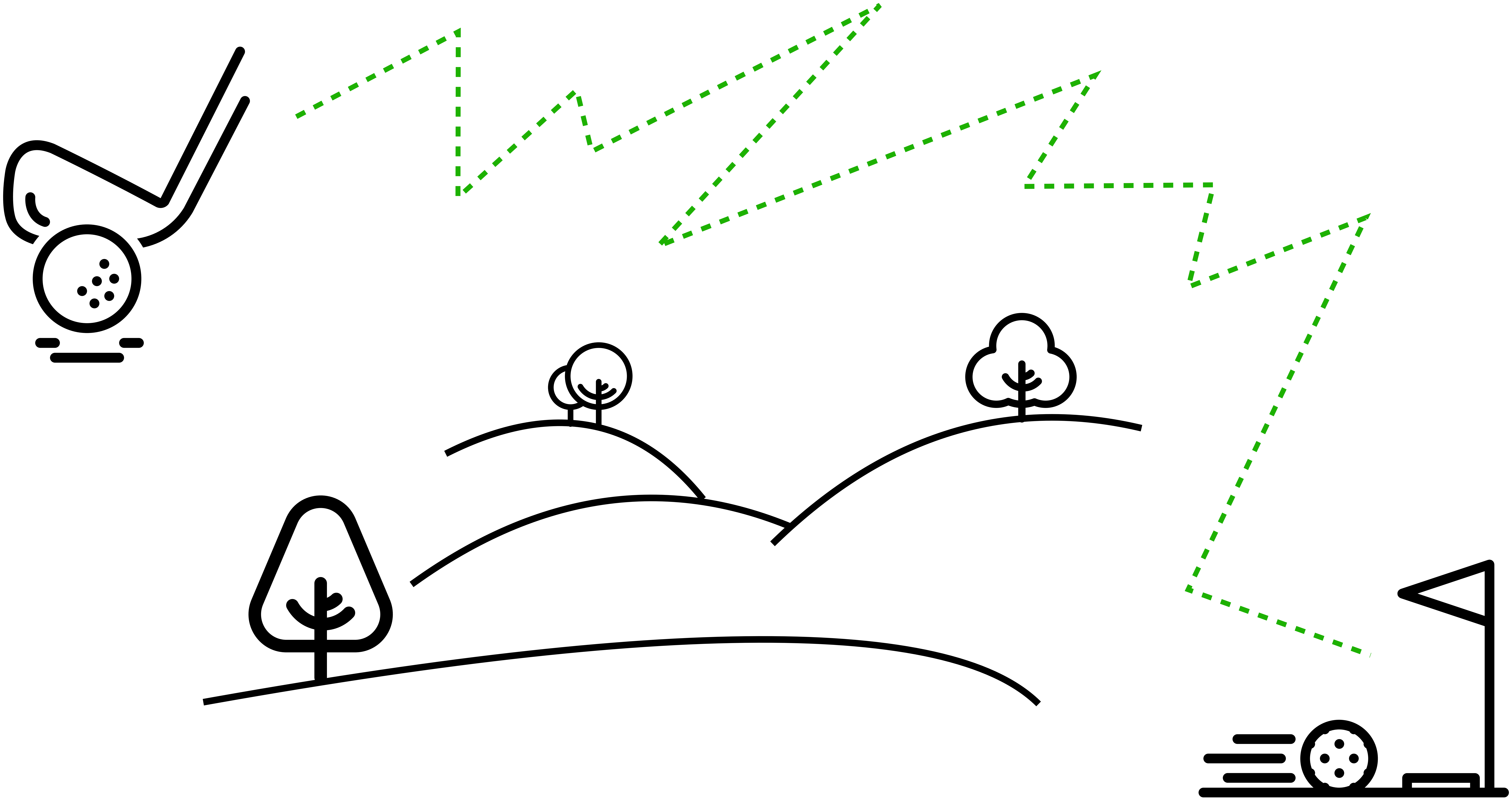
calculus of variations

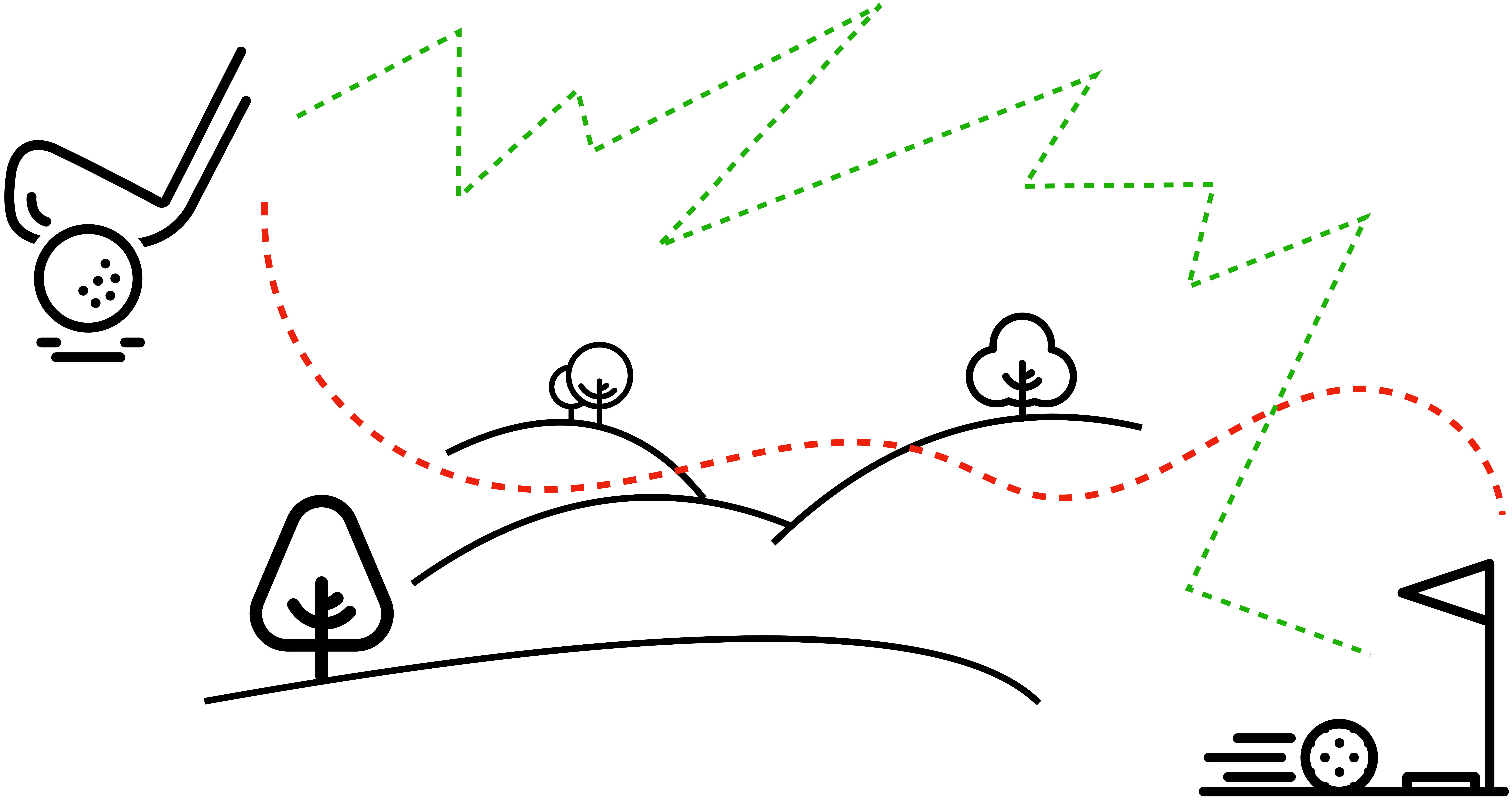
optimal control

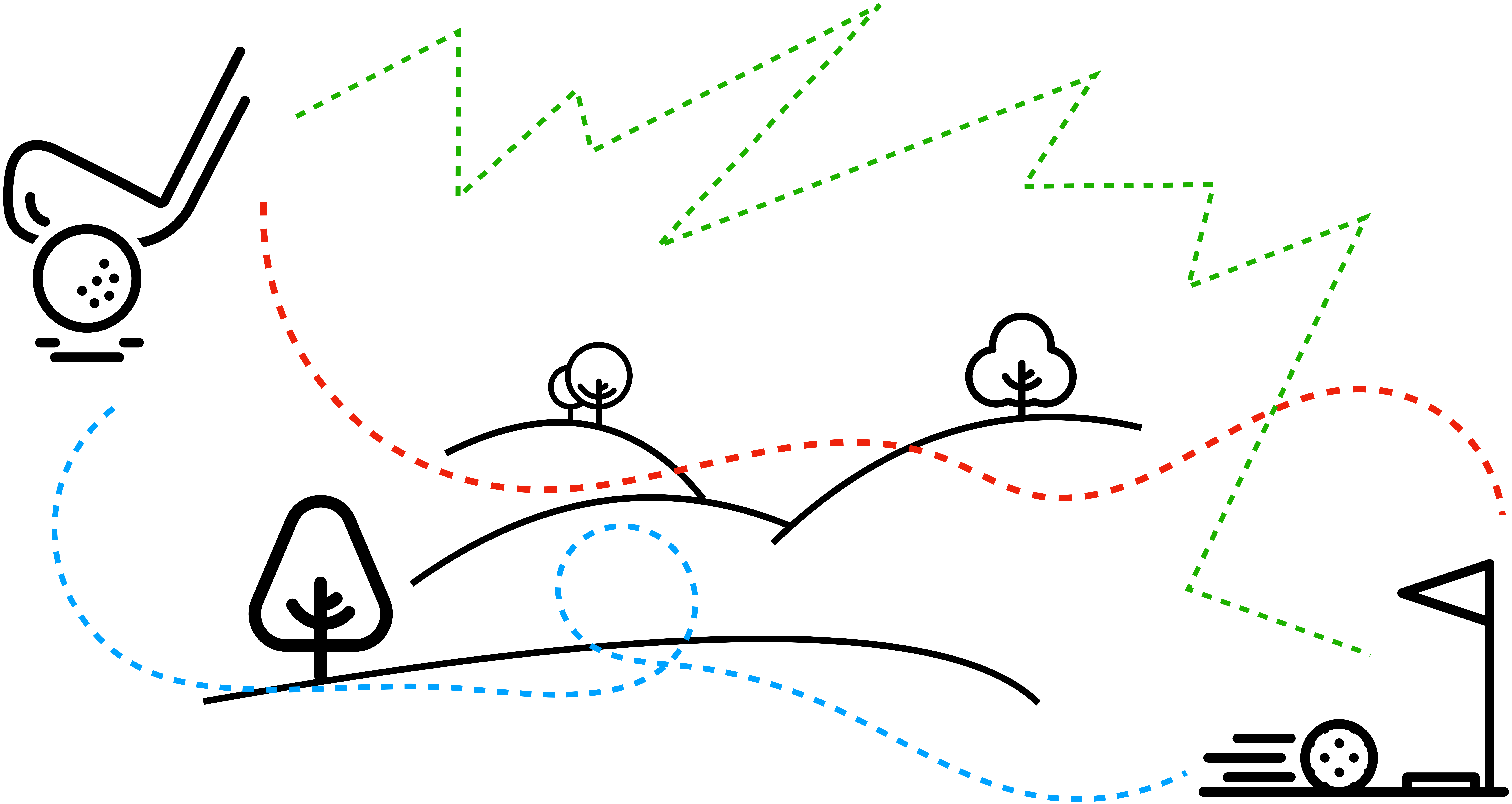
dynamic programming


EUROPEAN
TOUR


EUROPEAN
TOUR







how does the ball *know* where to roll??

how does the ball *know* where to roll??

It doesn't. It just follows Newton's laws at every instant in time

$$F=ma$$

how does the ball *know* where to roll??

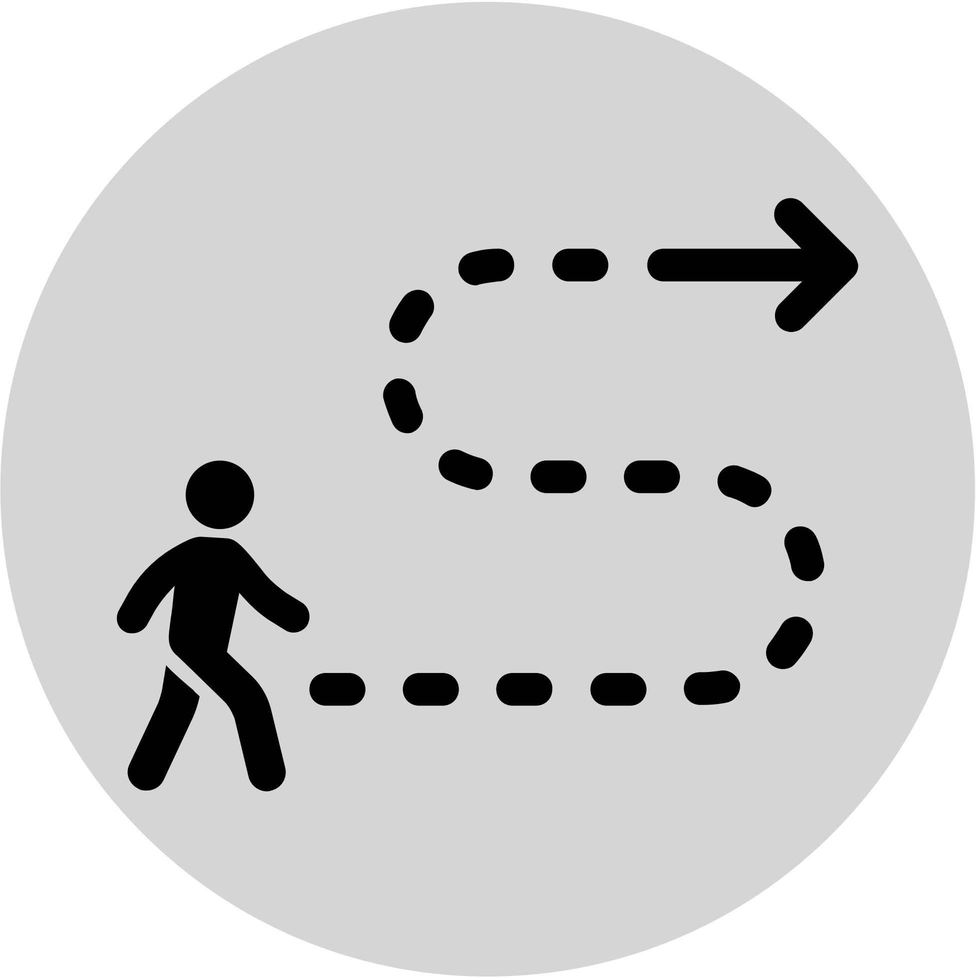
It doesn't. It just follows Newton's laws at every instant in time

F=ma

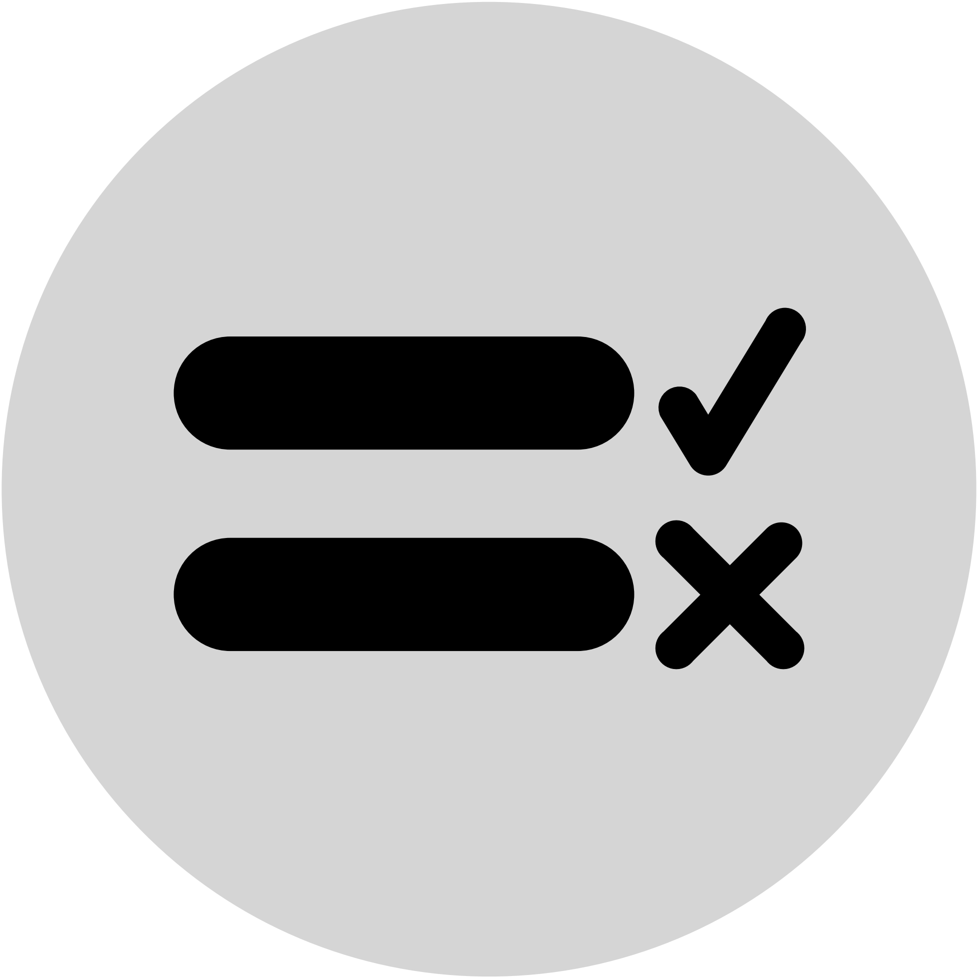
Every path has a score called "action". The actual path is the one with the lowest score

$$\int \mathscr{L} \, dt$$

observed path

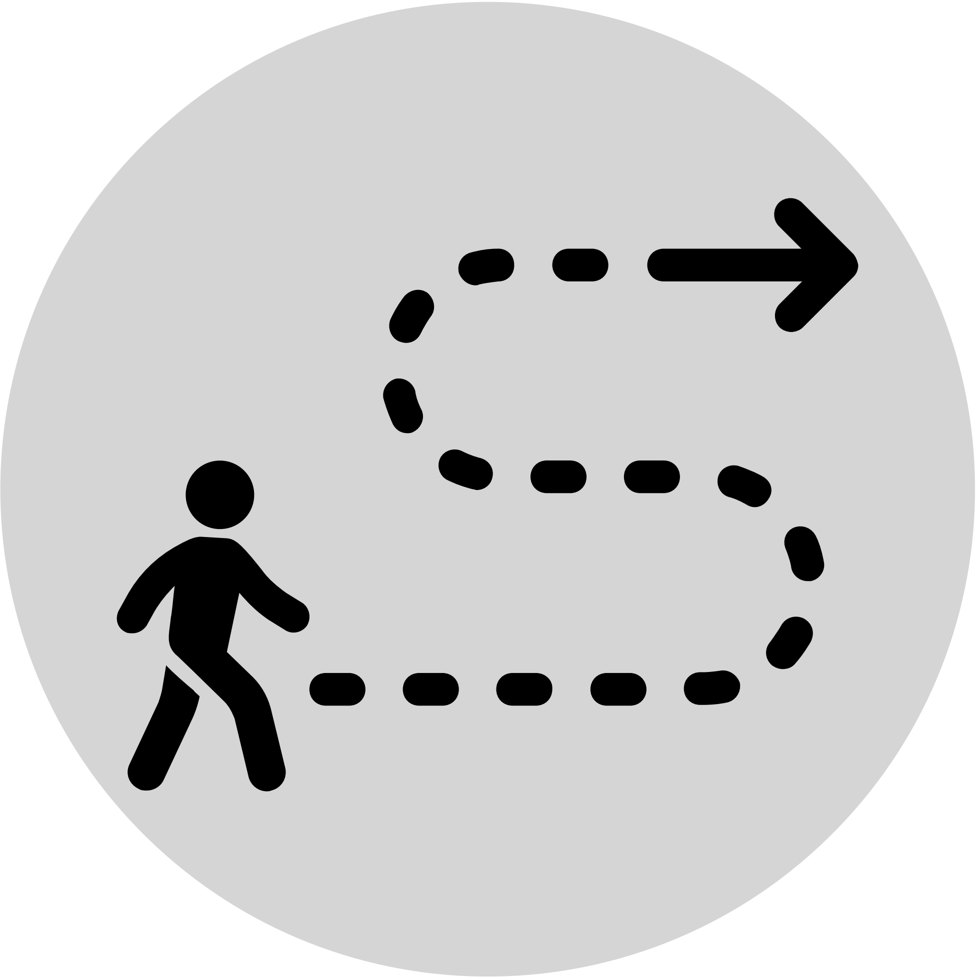


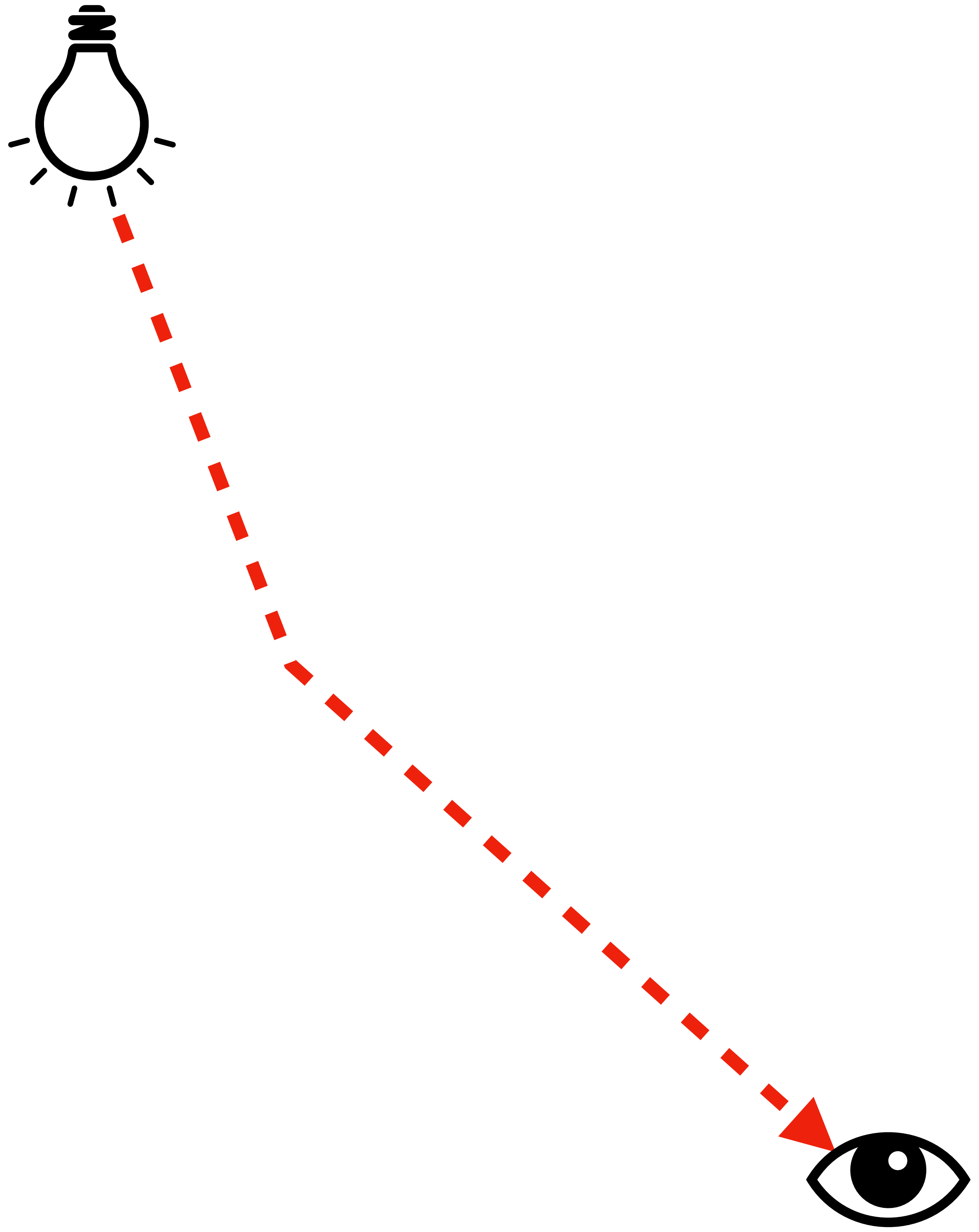
instantaneous rule



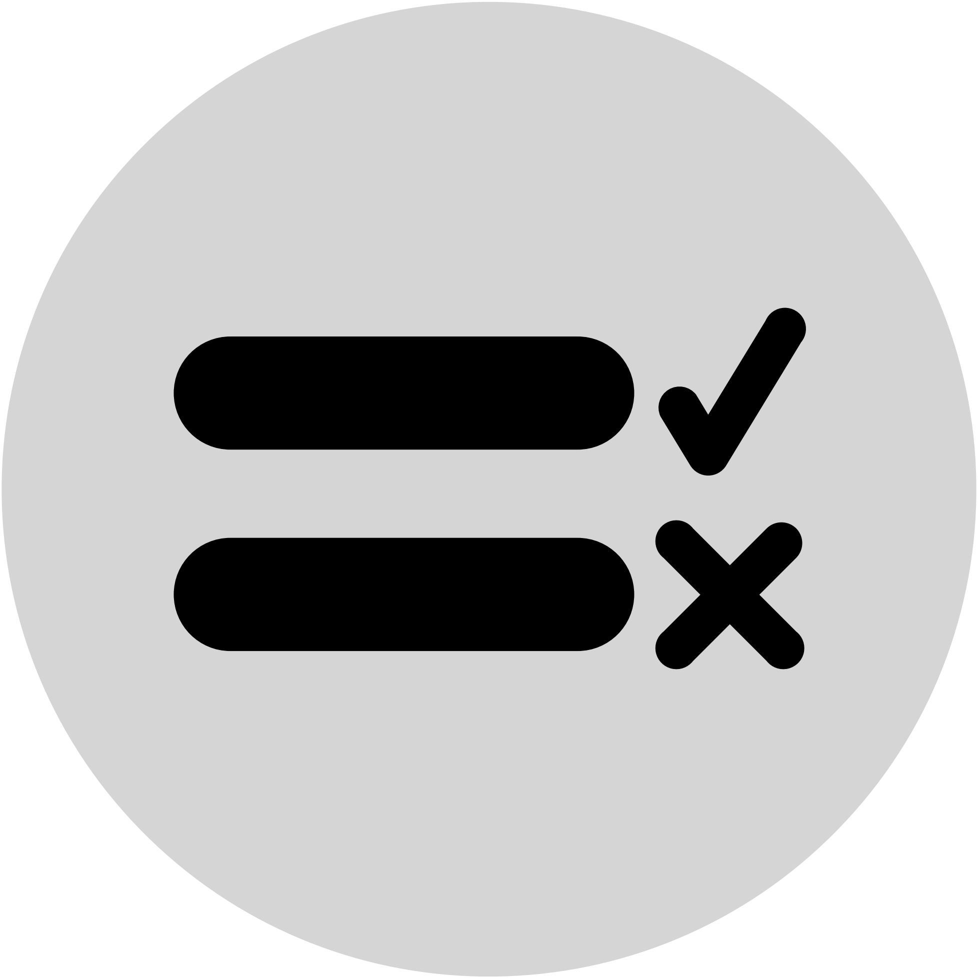
global principle

observed path





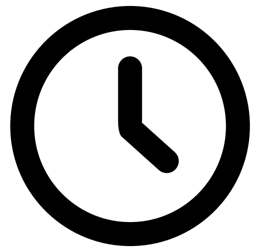
instantaneous rule



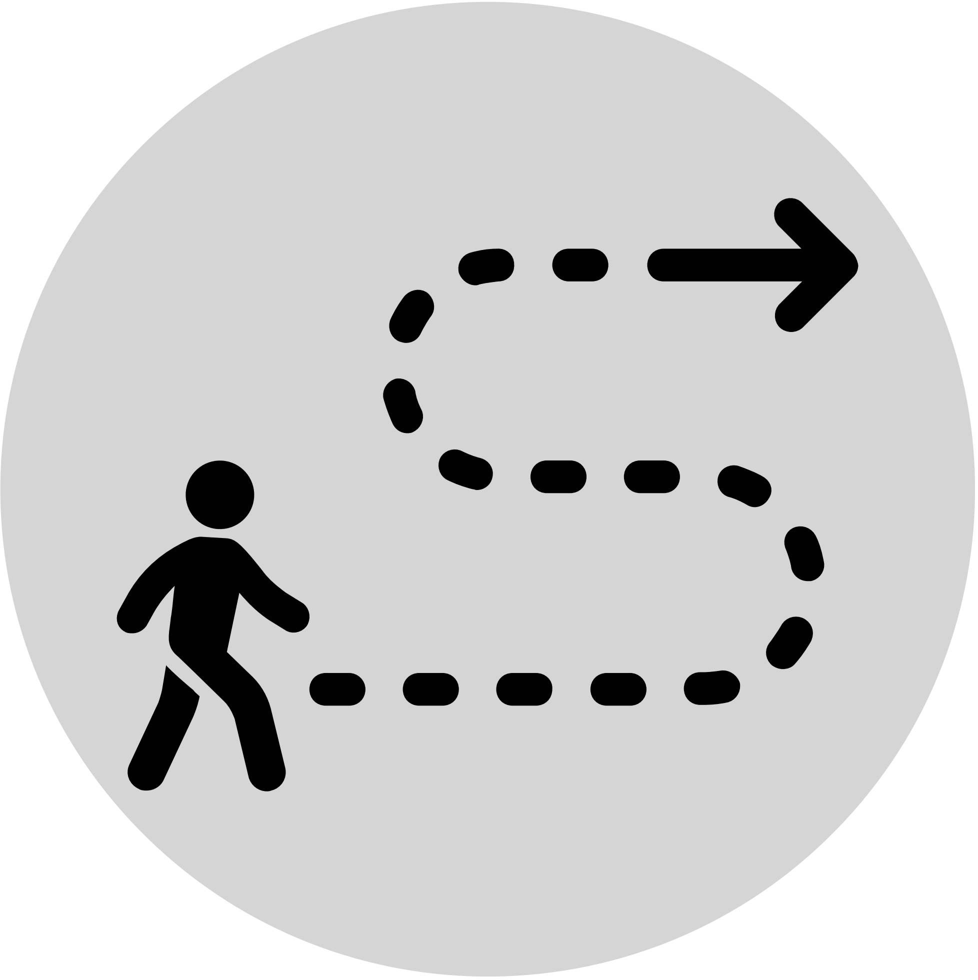
$$\frac{\sin(\theta_1)}{v_1} = \frac{\sin(\theta_2)}{v_2}$$

global principle

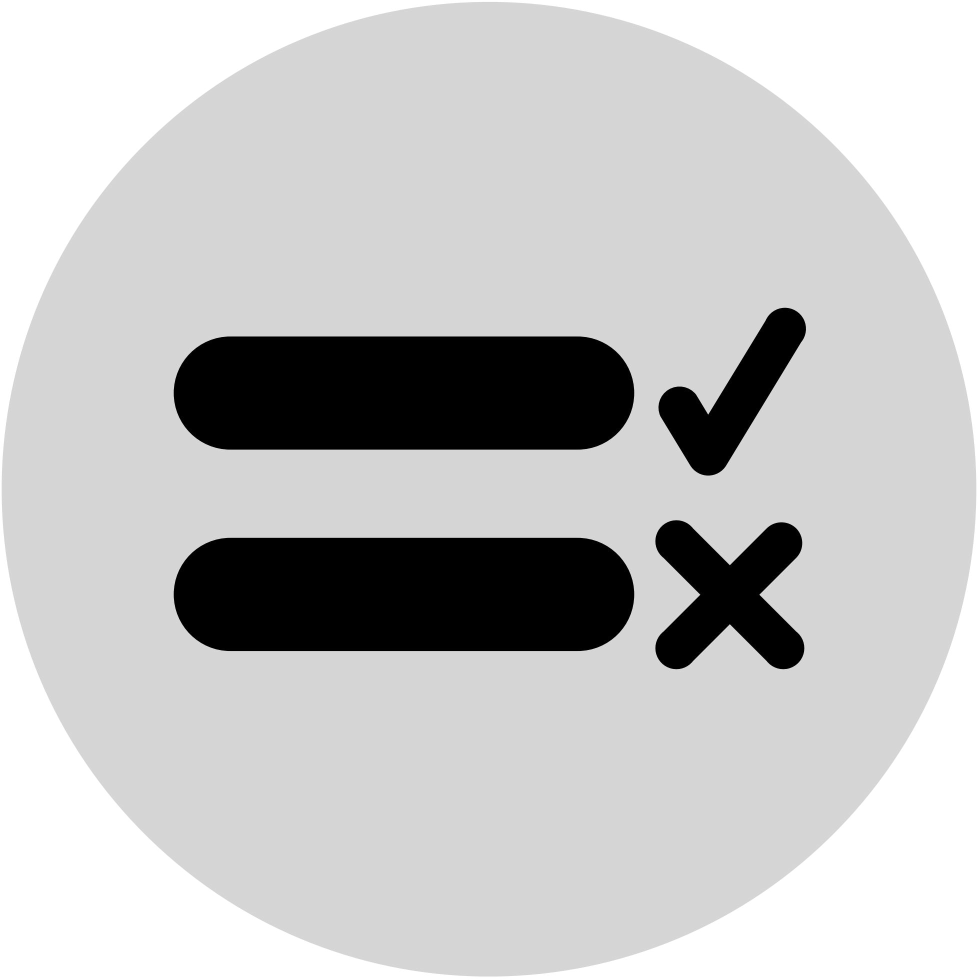
min time



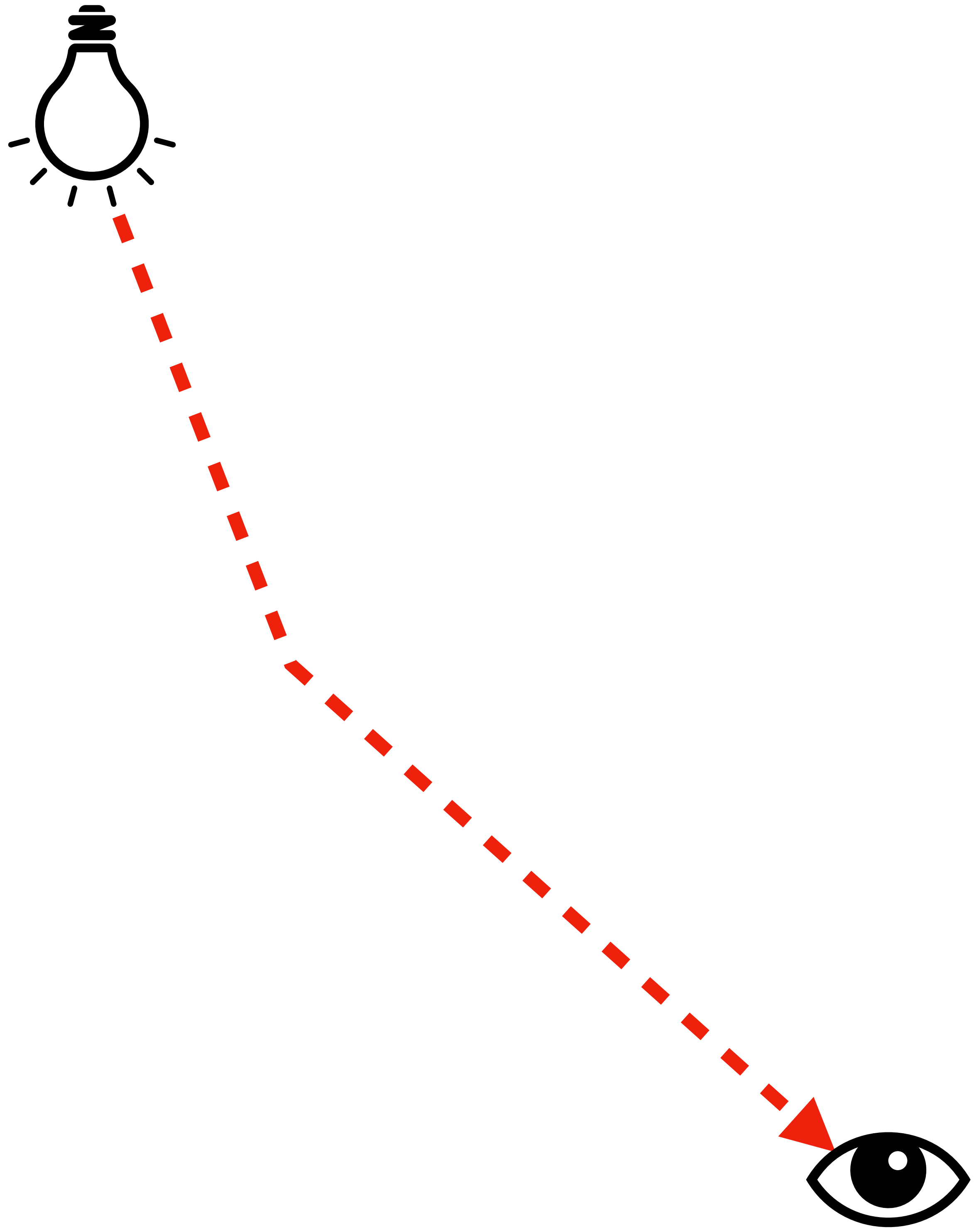
observed path



instantaneous rule

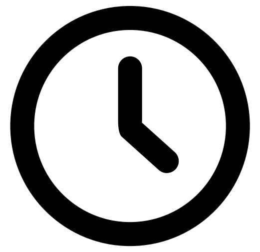


global principle



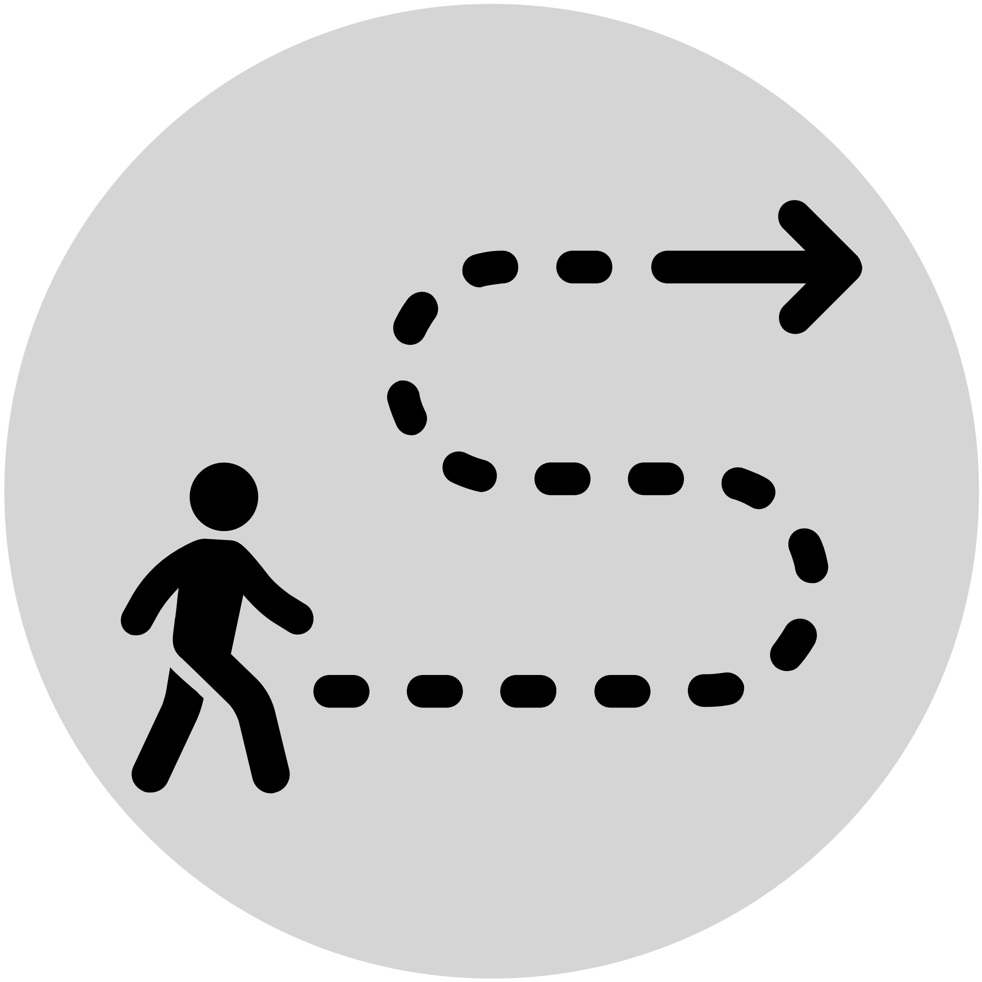
$$\frac{\sin(\theta_1)}{v_1} = \frac{\sin(\theta_2)}{v_2}$$

min time

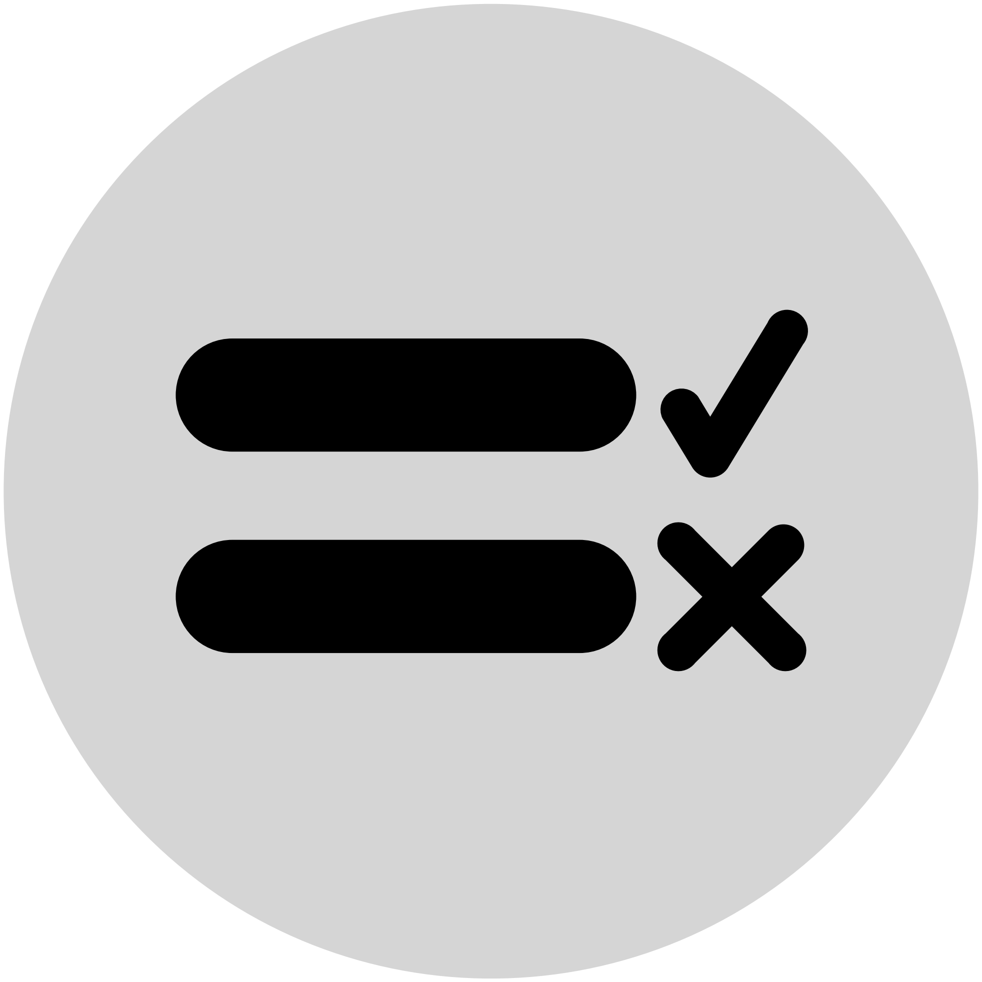


Fermat's principle

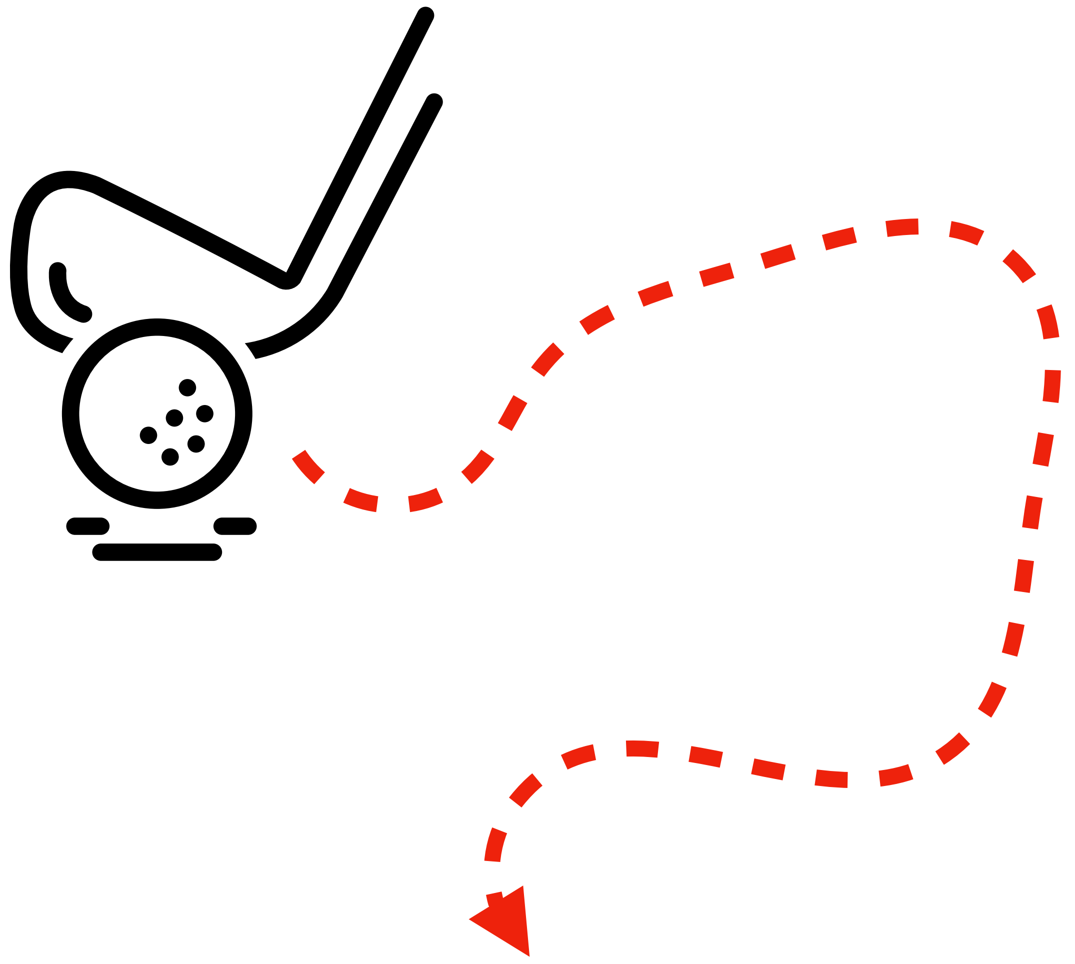
observed path



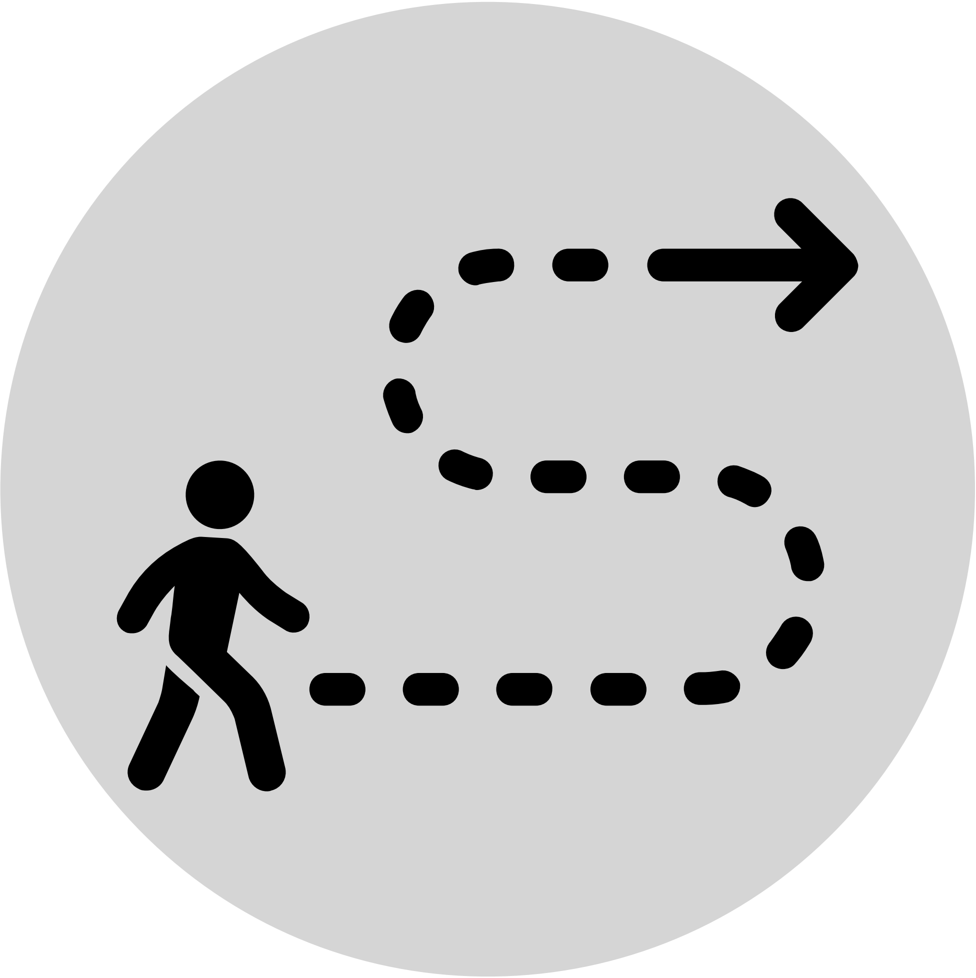
instantaneous rule

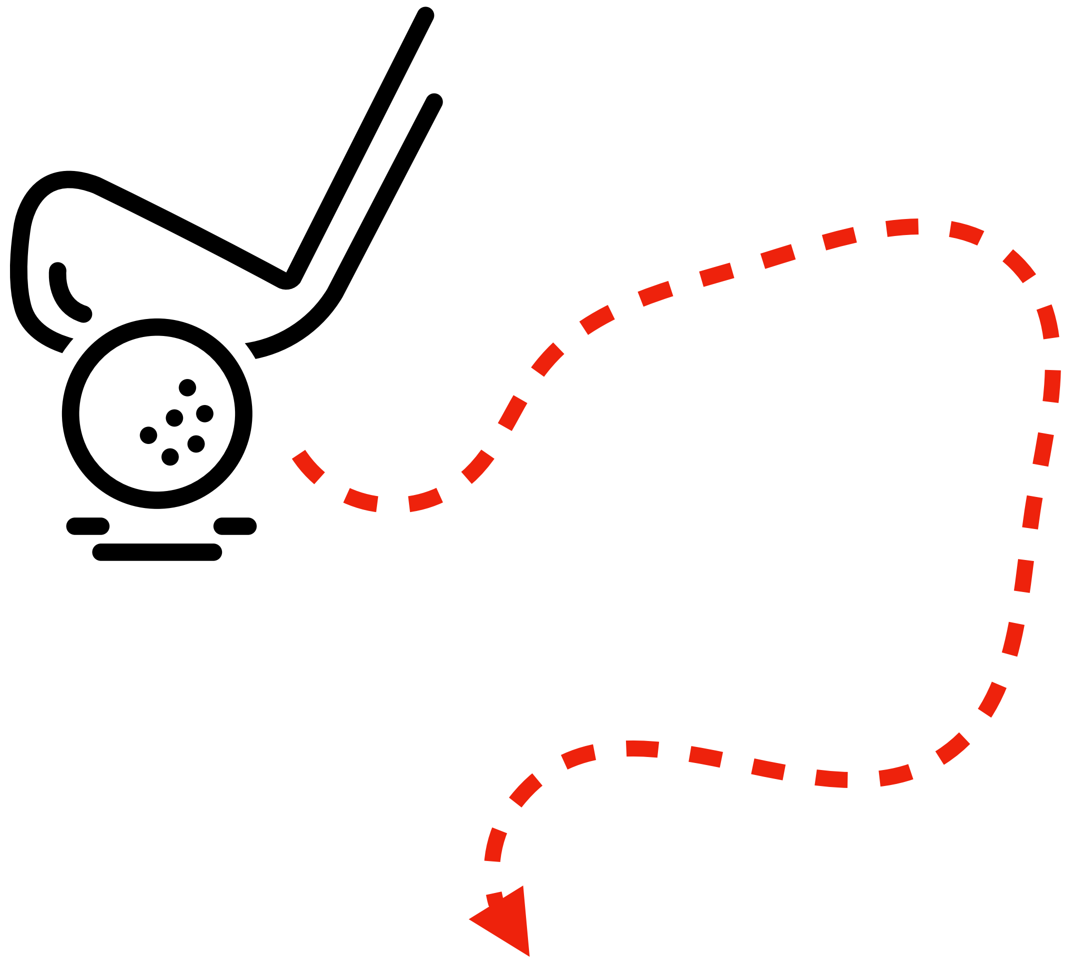


global principle

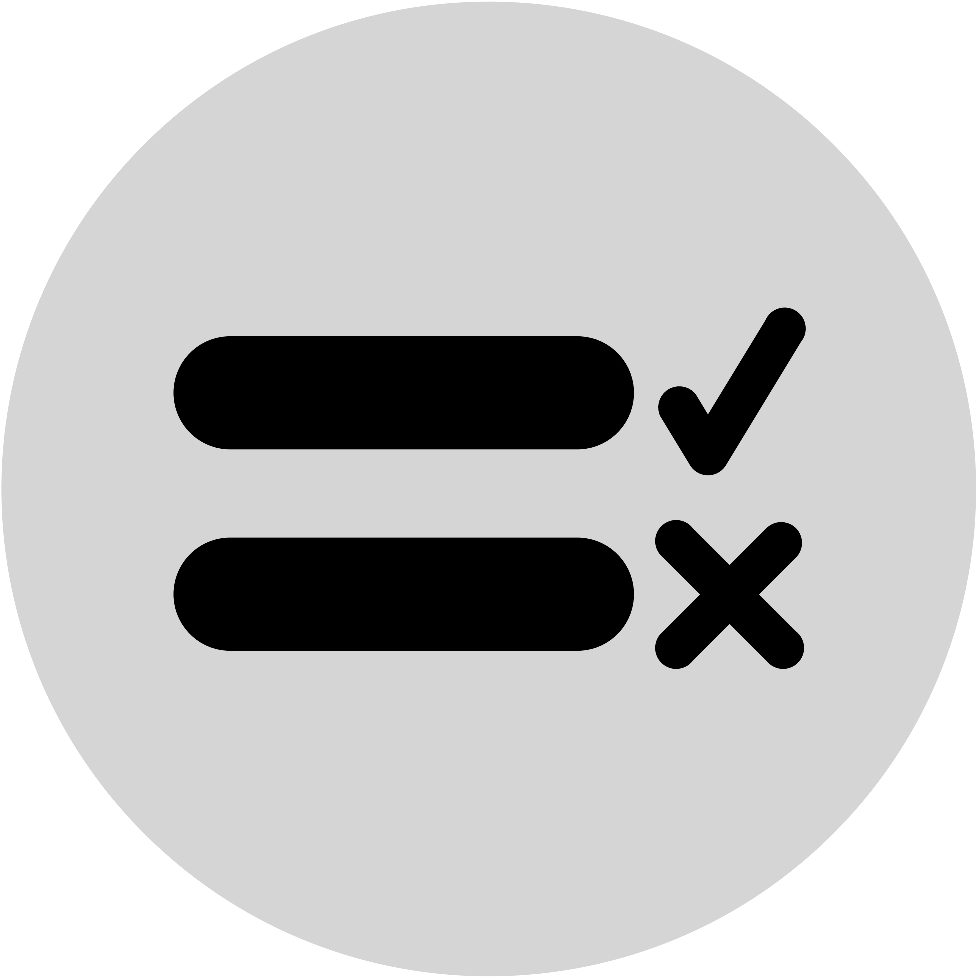


observed path





instantaneous rule



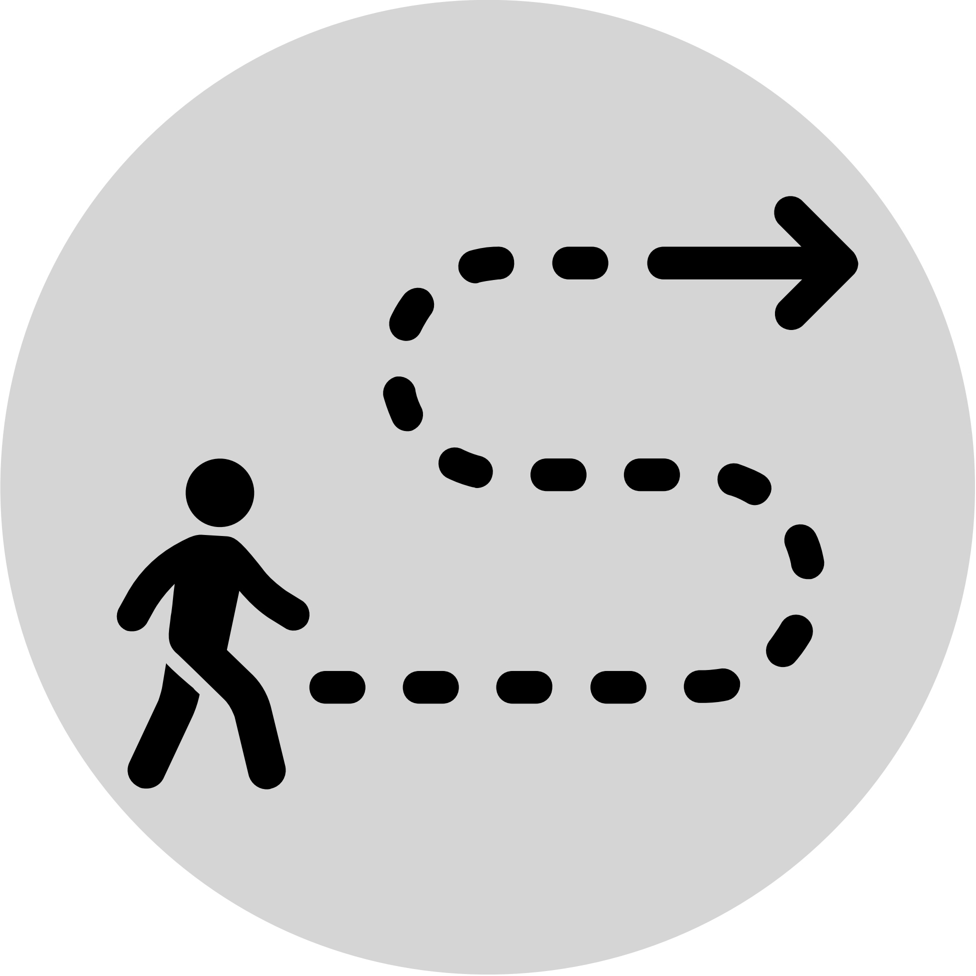
F=ma

global principle

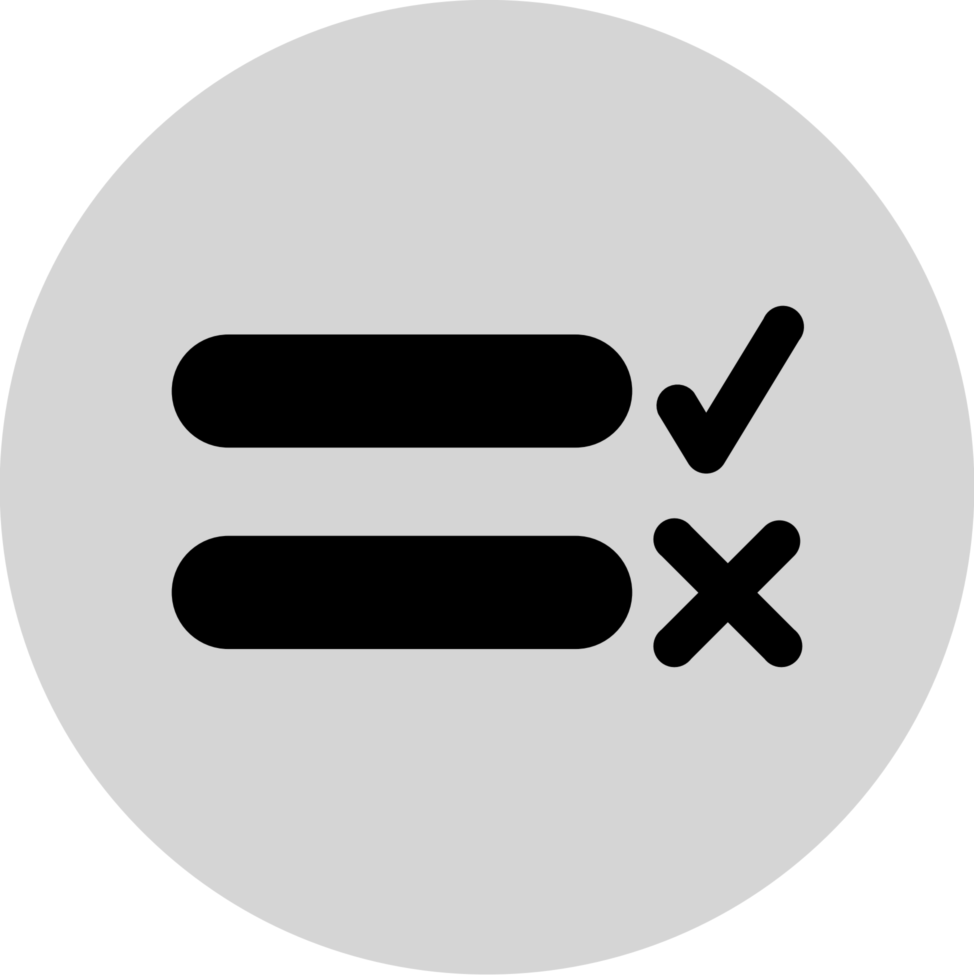
min action

$$\int \mathscr{L}\, dt$$

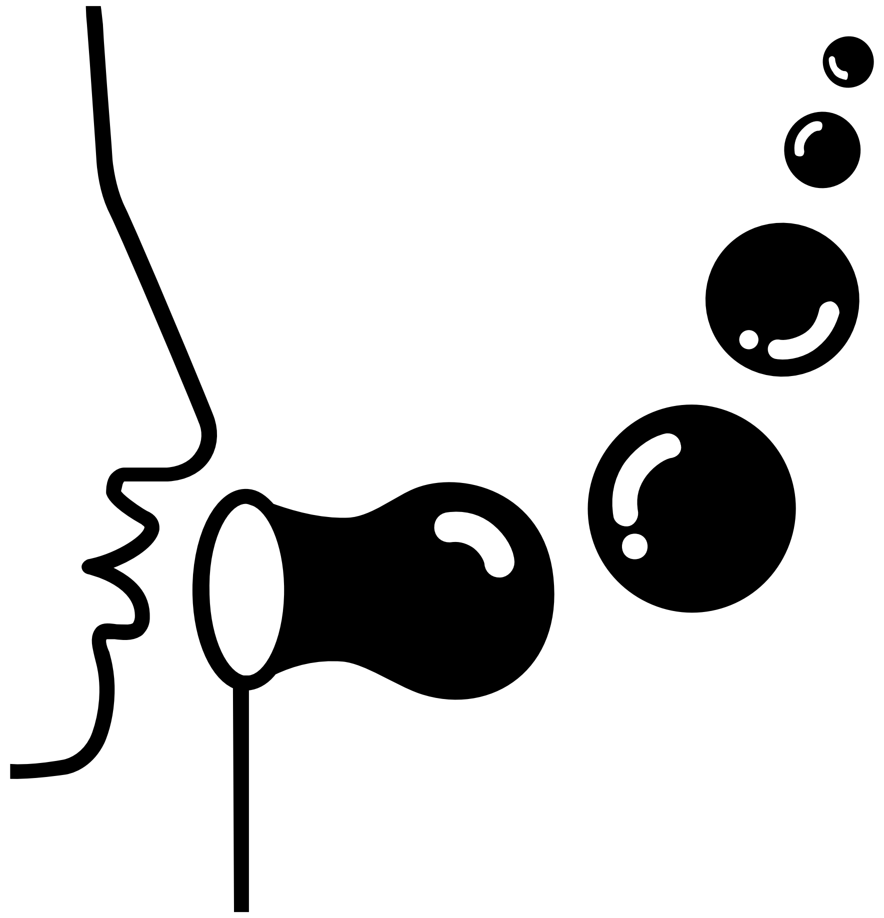
observed path



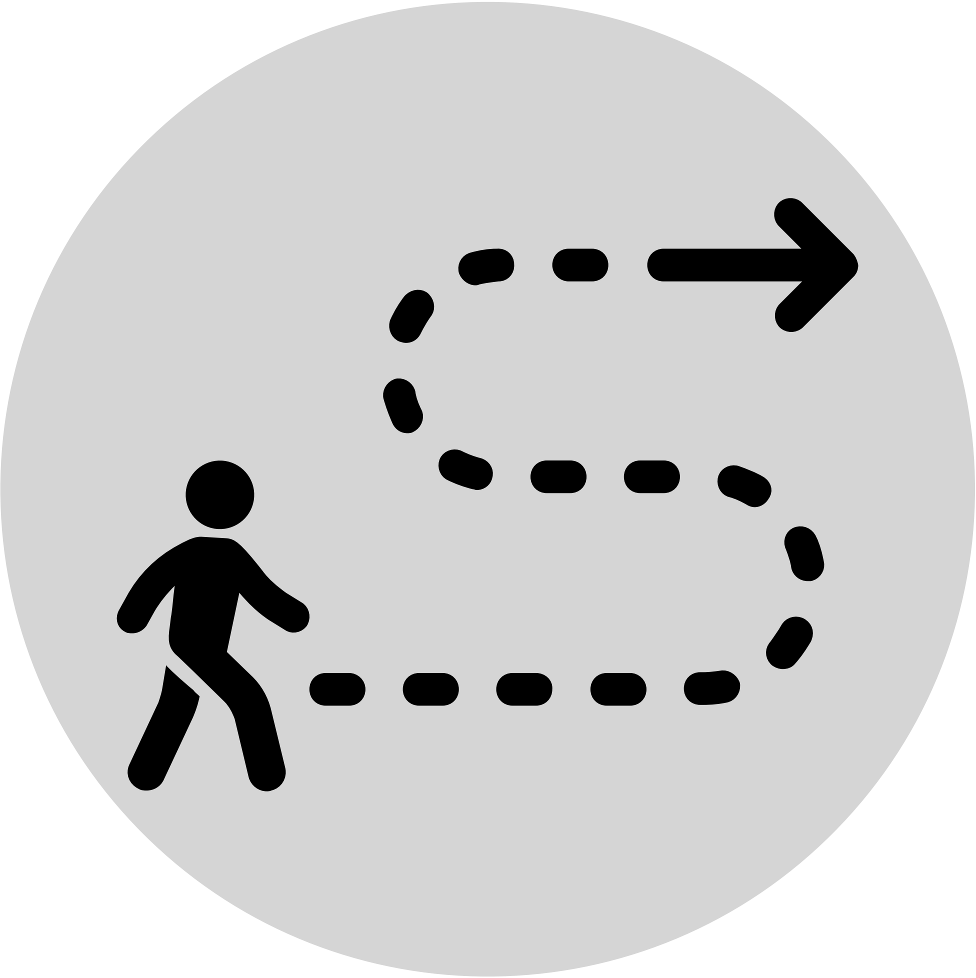
instantaneous rule



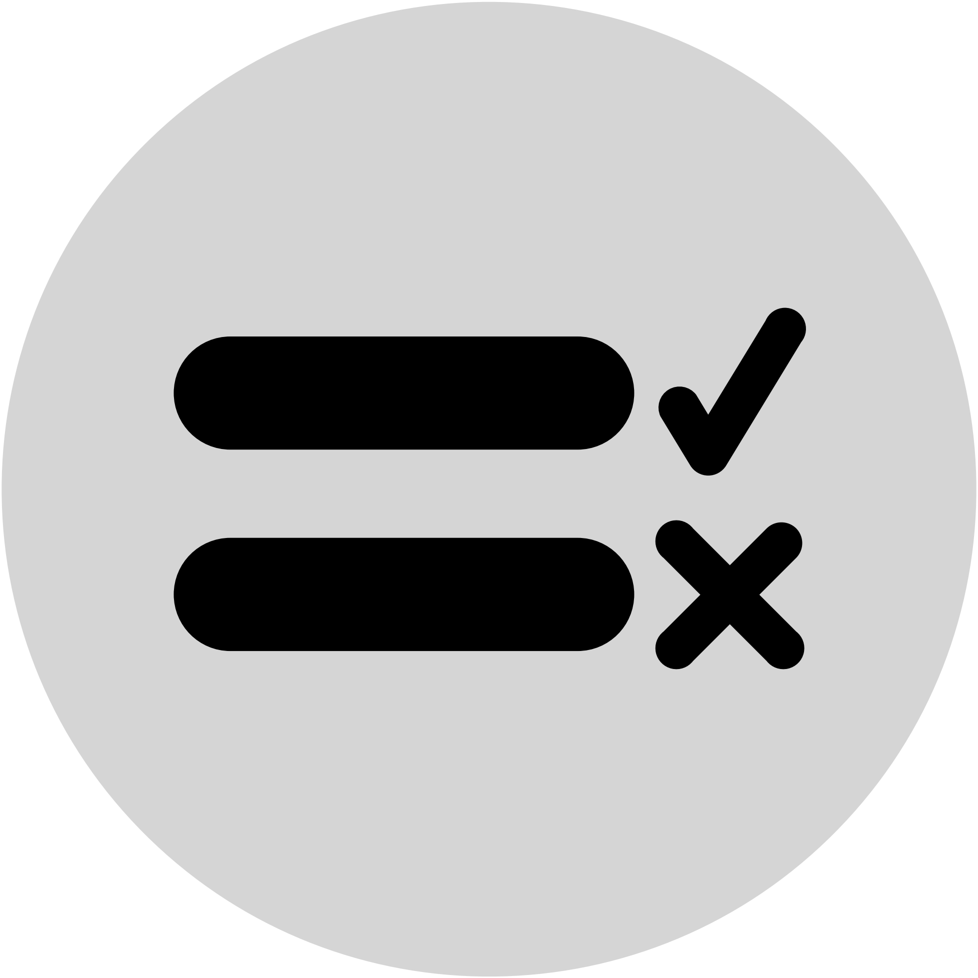
global principle



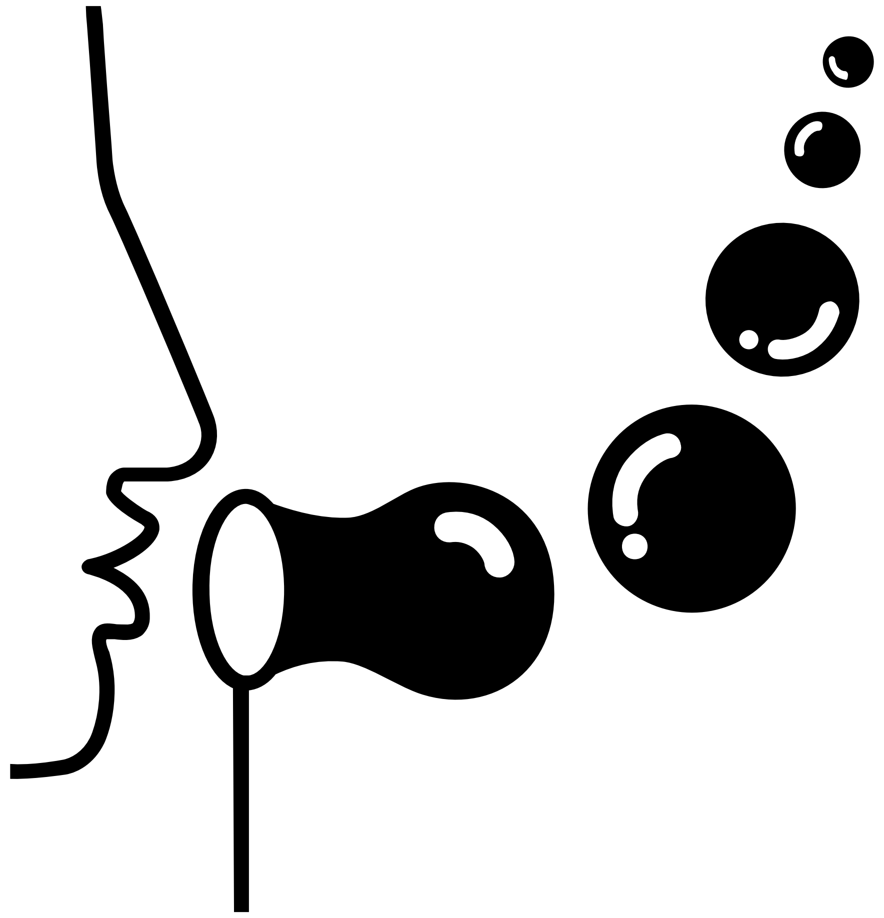
observed path



instantaneous rule



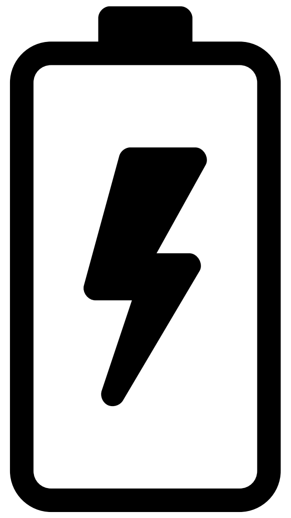
global principle



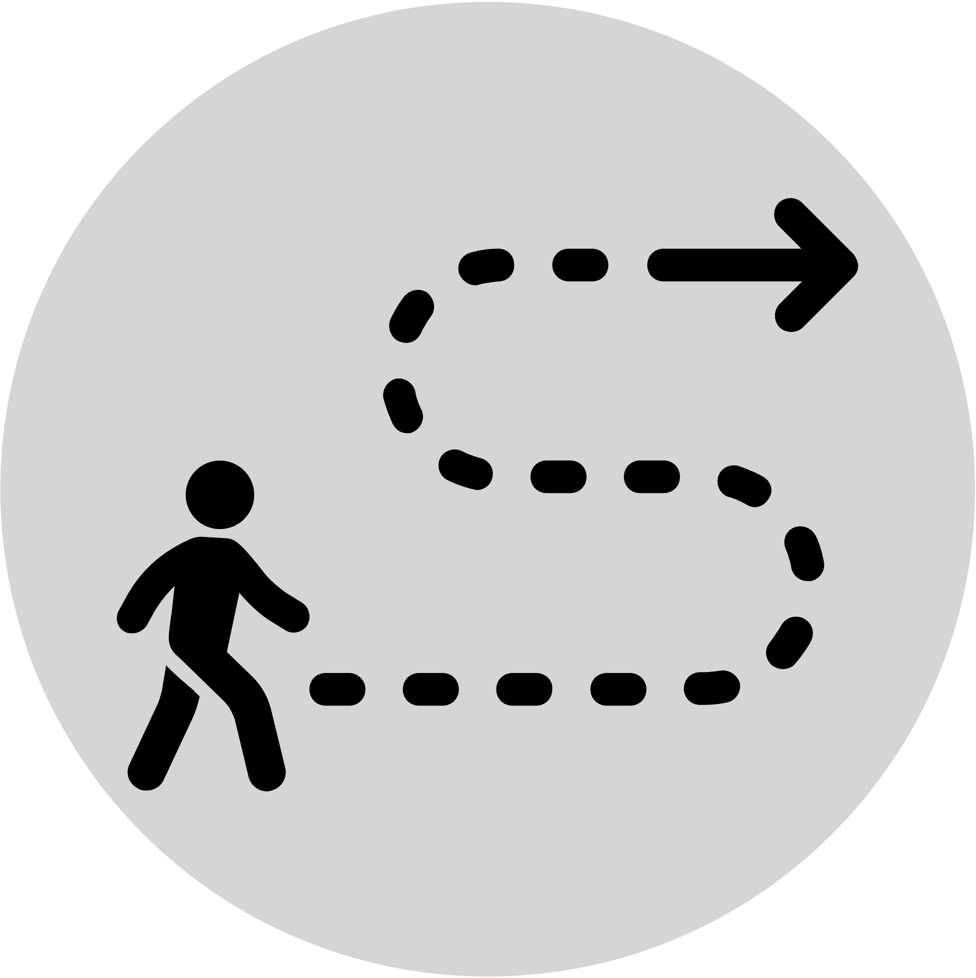
$$\Delta p = \frac{4\gamma}{R}$$

Young-Laplace

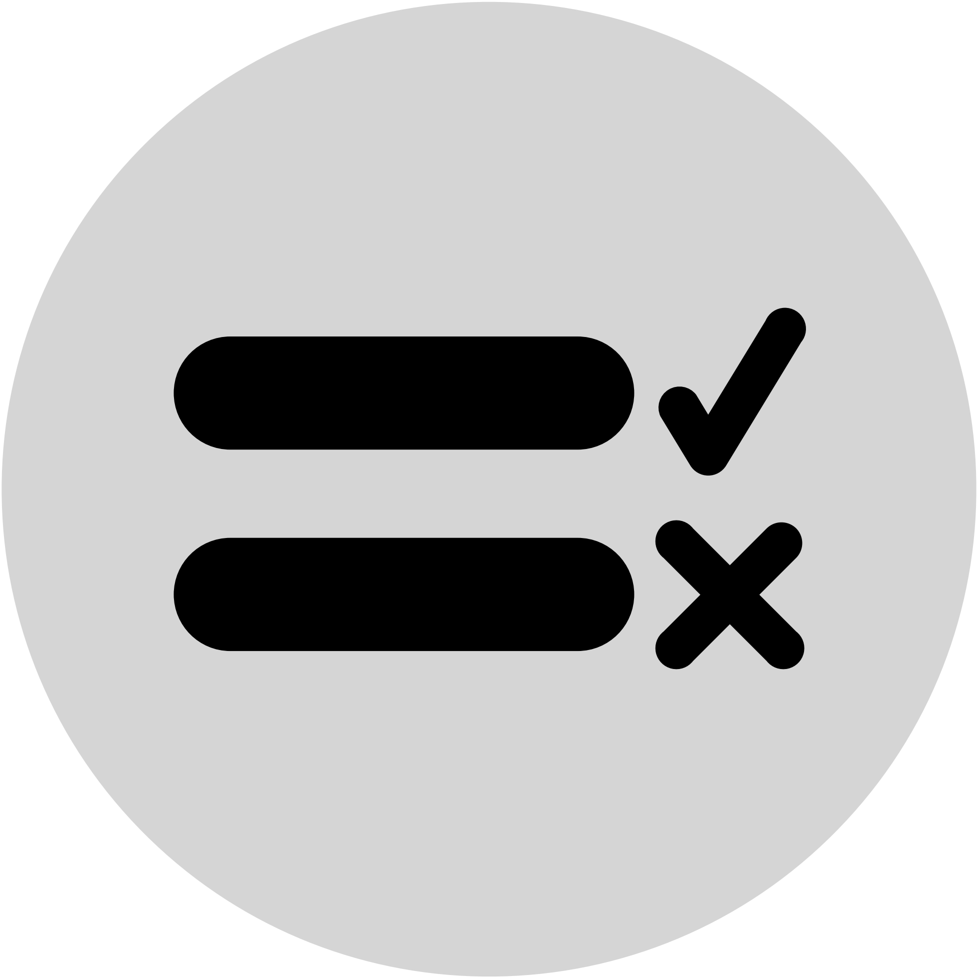
min energy



observed path

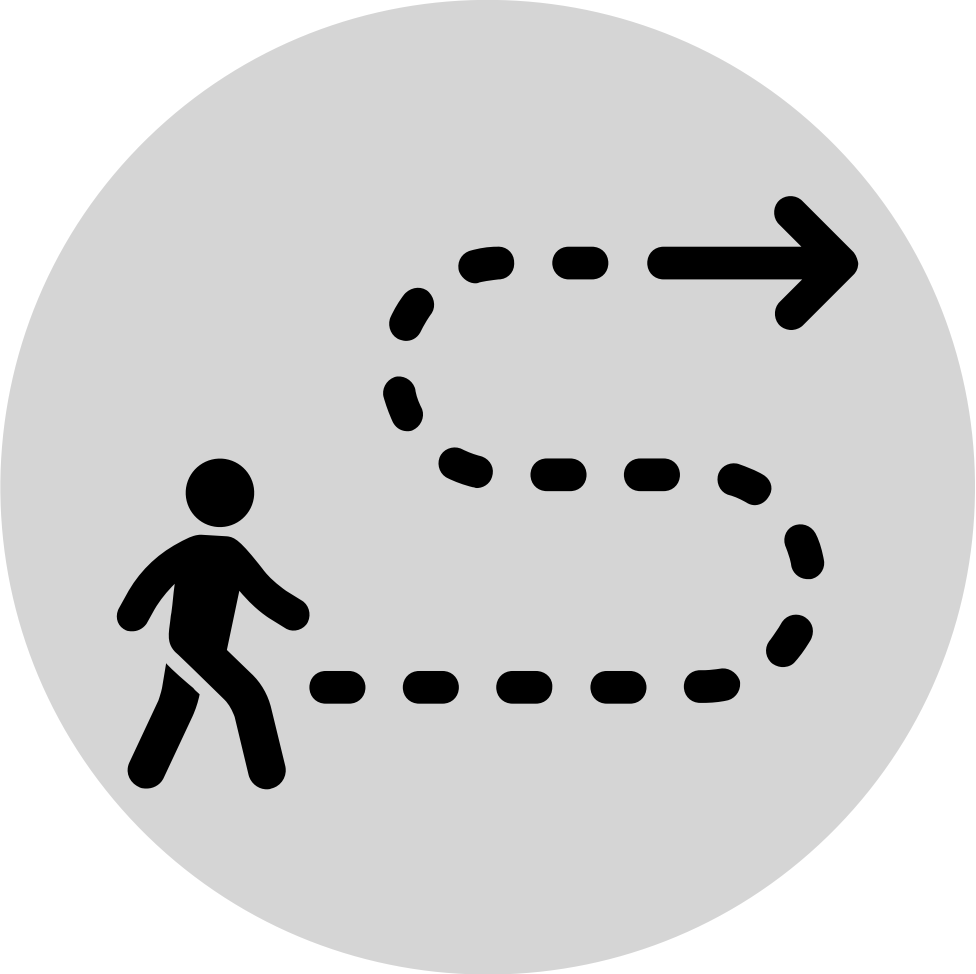


instantaneous rule

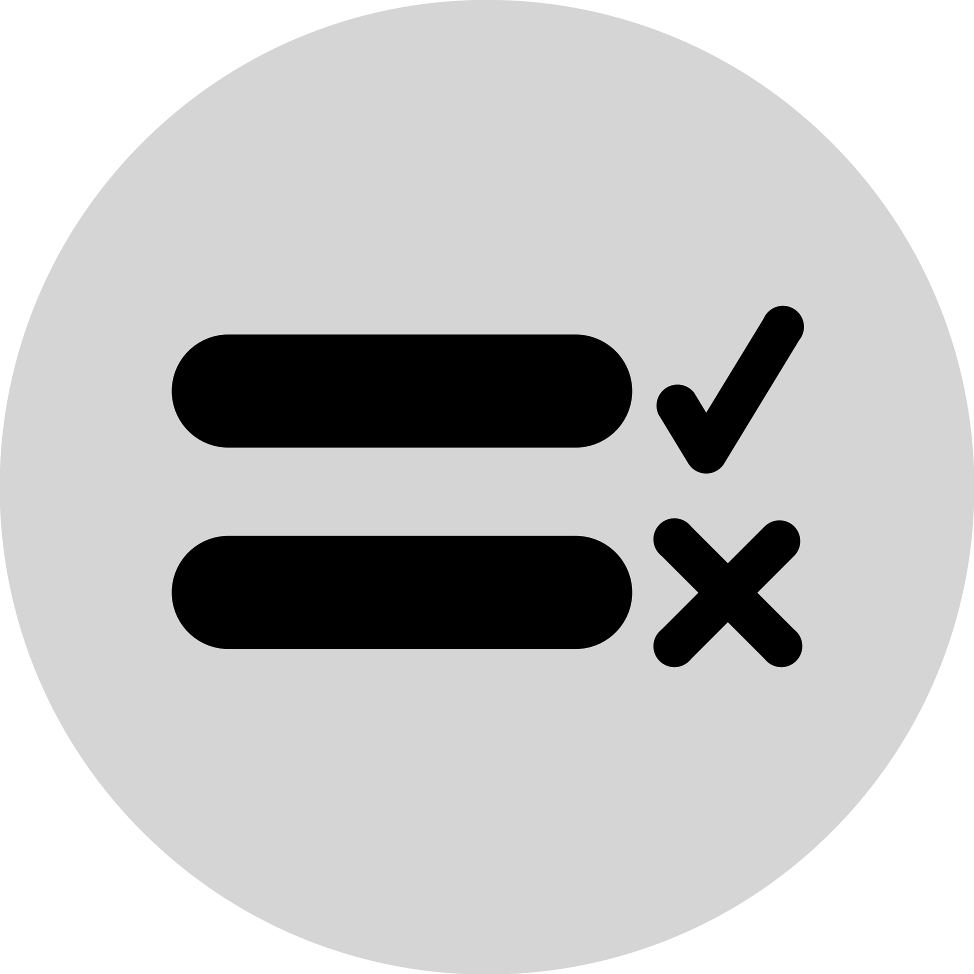


global principle

observed path



instantaneous rule

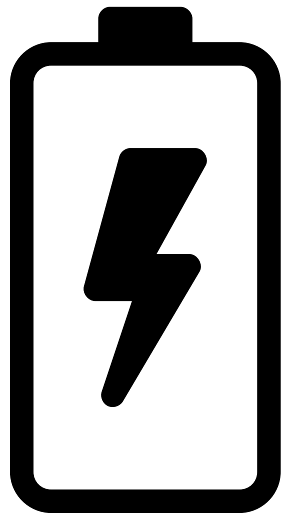


global principle

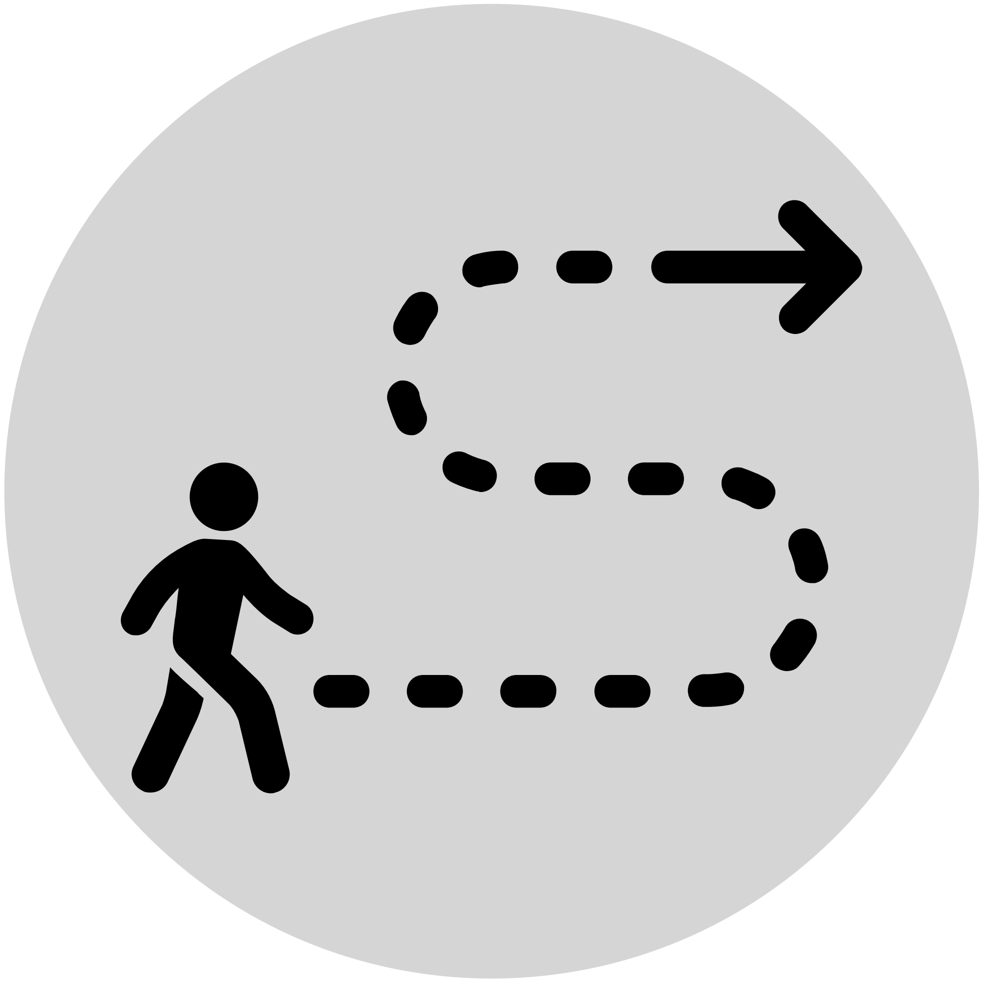
$$y = \frac{1}{a}\cosh(ax)$$

catenary

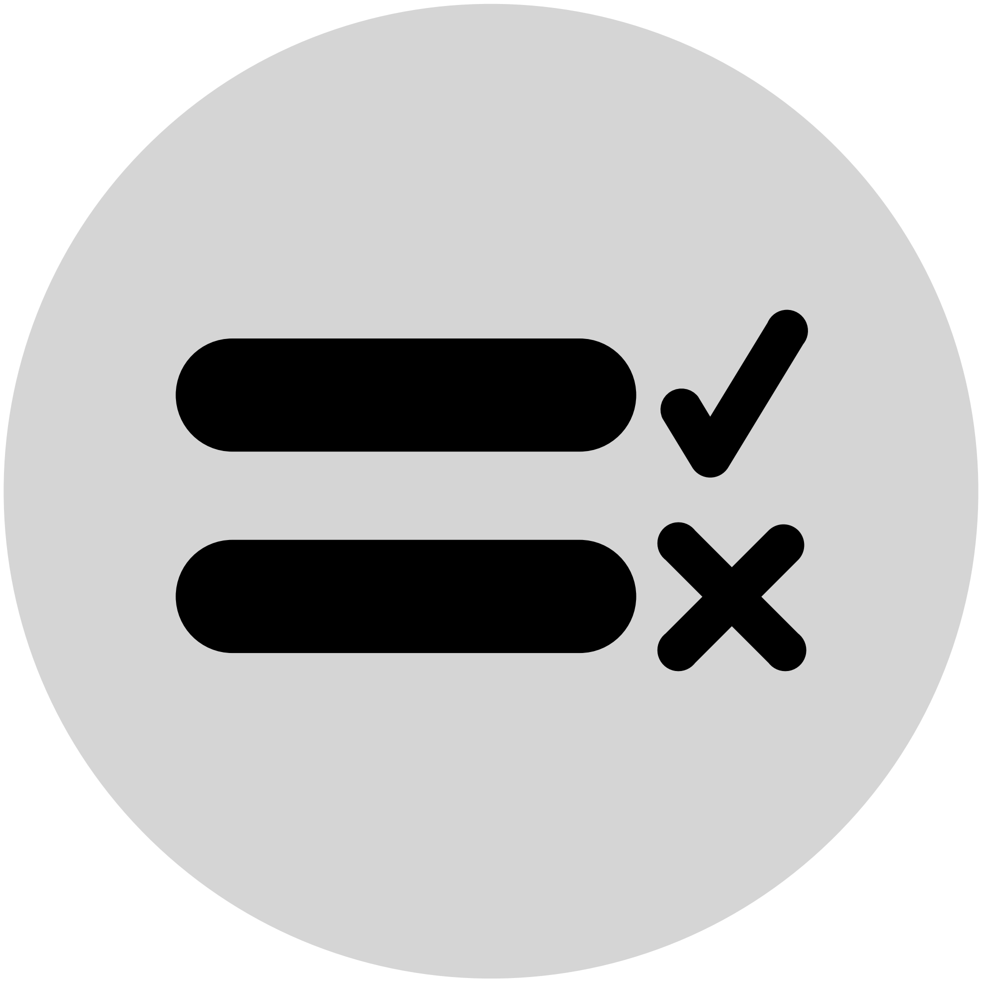
min potential



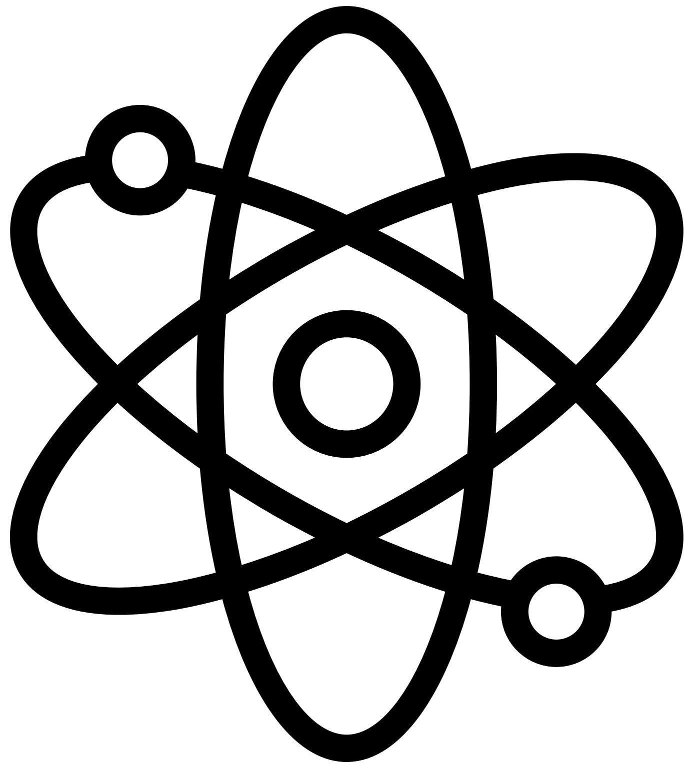
observed path



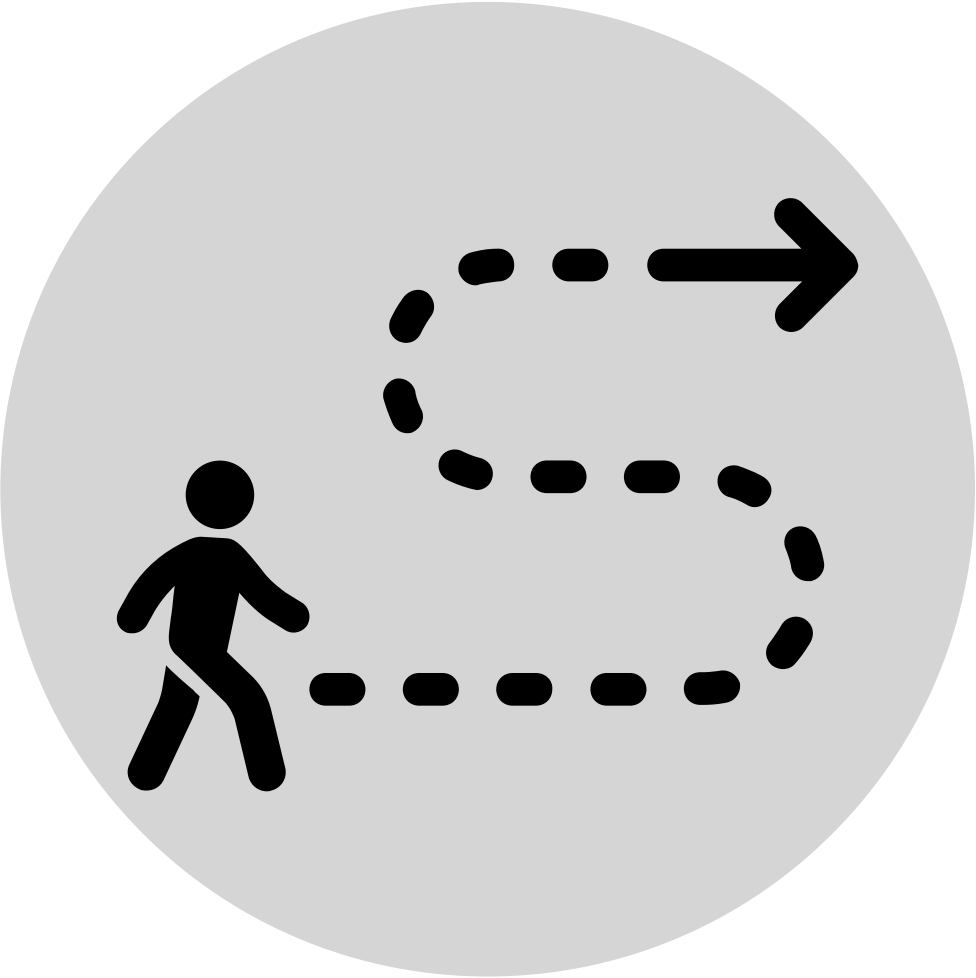
instantaneous rule



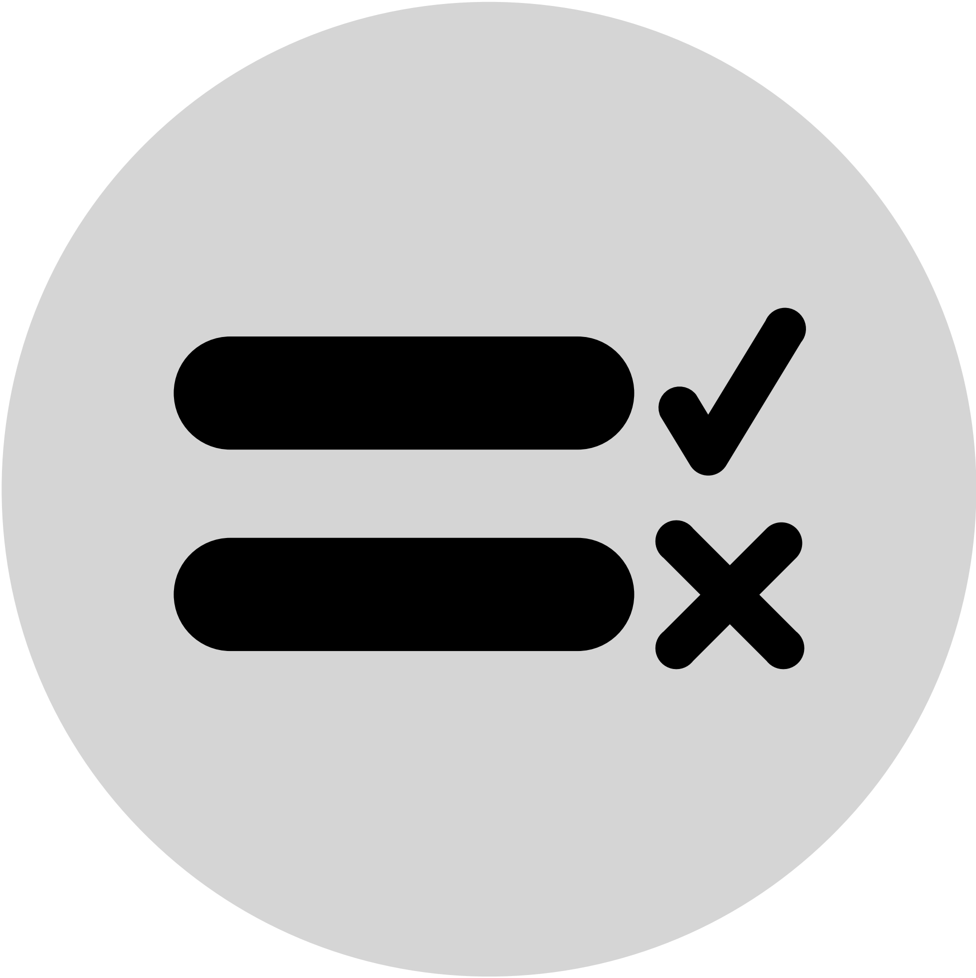
global principle



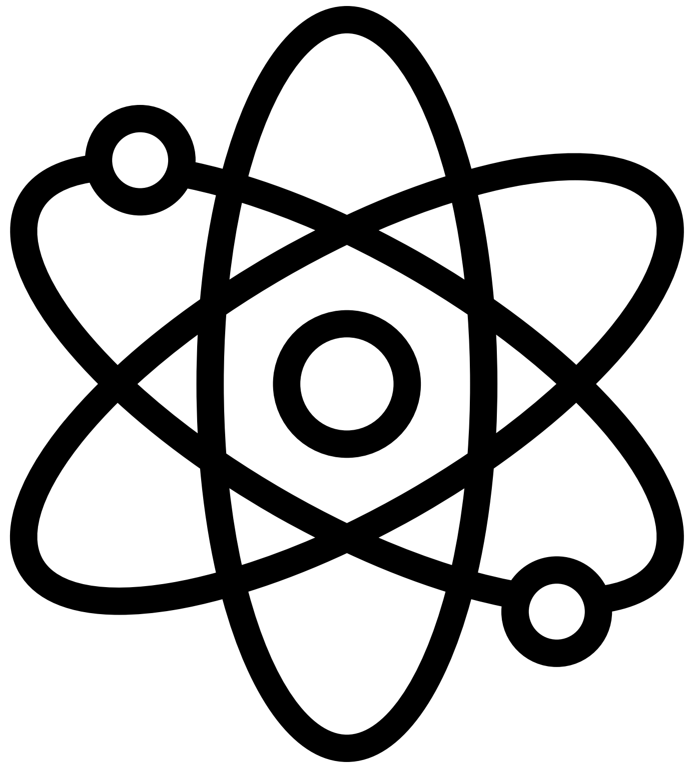
observed path



instantaneous rule



global principle



$$
\begin{aligned}
\mathcal{L} = & -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
& + i\bar{\psi}\not{D}\psi + h.c \\
& + \psi_i y_{ij}\psi_j\phi + h.c \\
& + |D_\mu\phi|^2 - V(\phi)
\end{aligned}
$$

Standard Model Formula

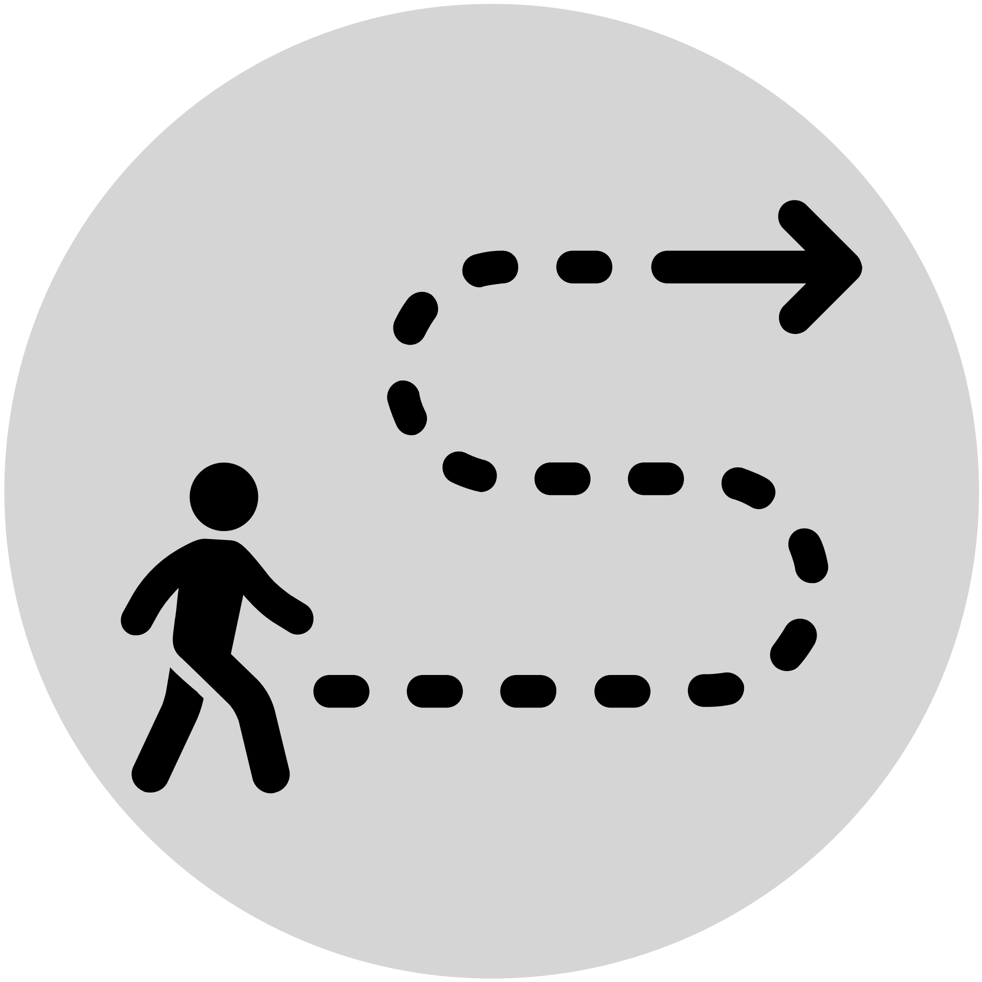
min action

$$\int \mathscr{L}\, d^n s$$

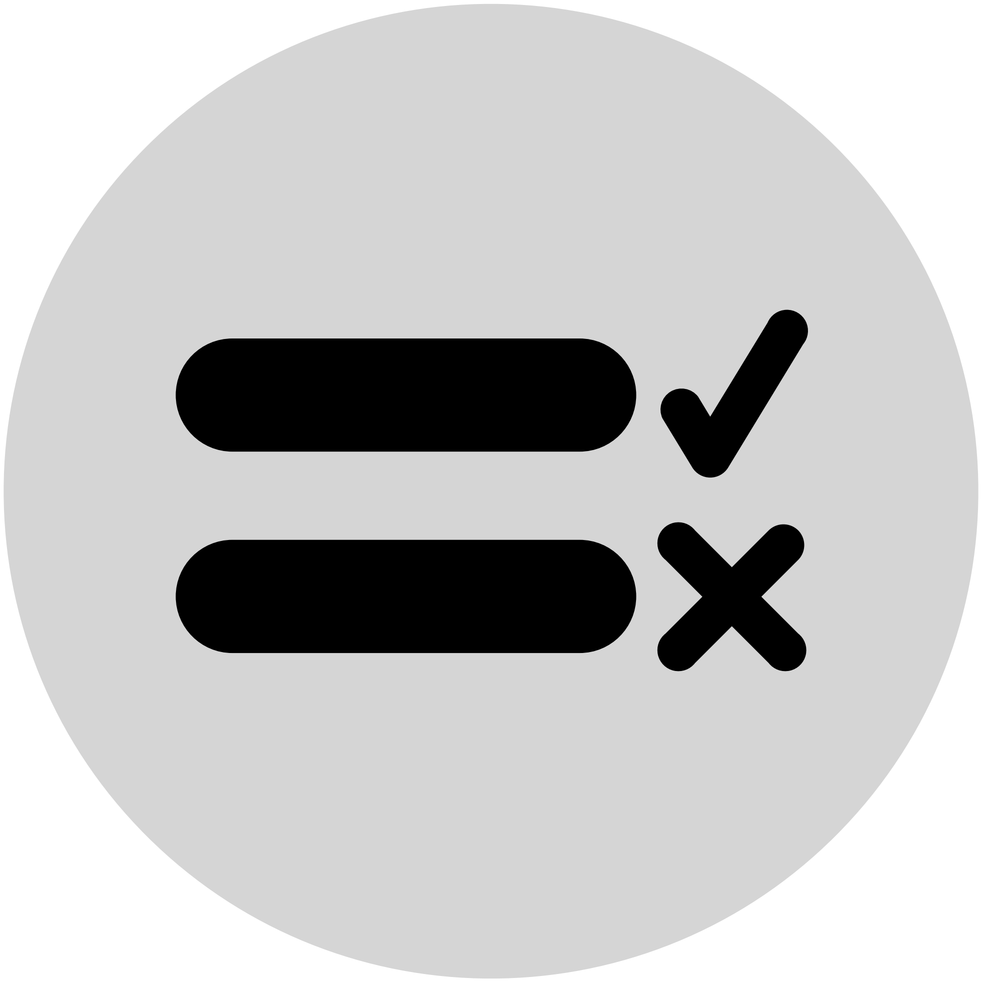
Optimal stomatal control

Yair Mau

observed path

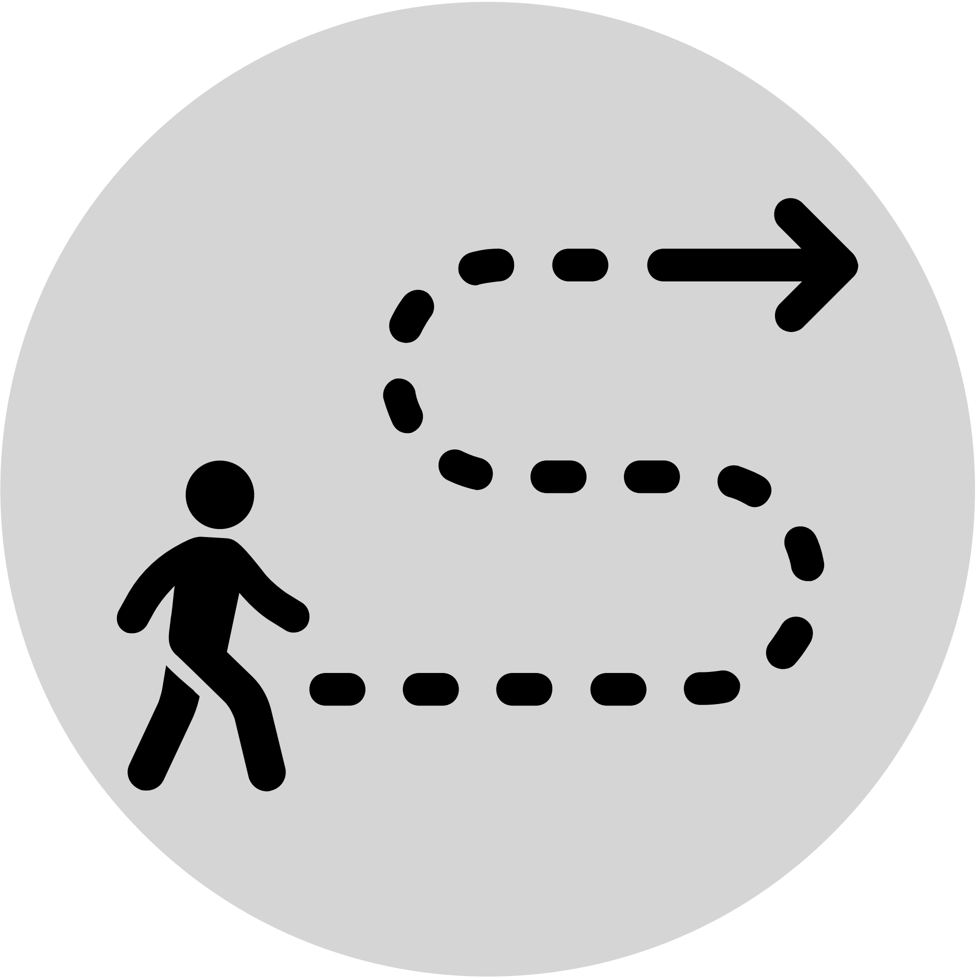


instantaneous rule

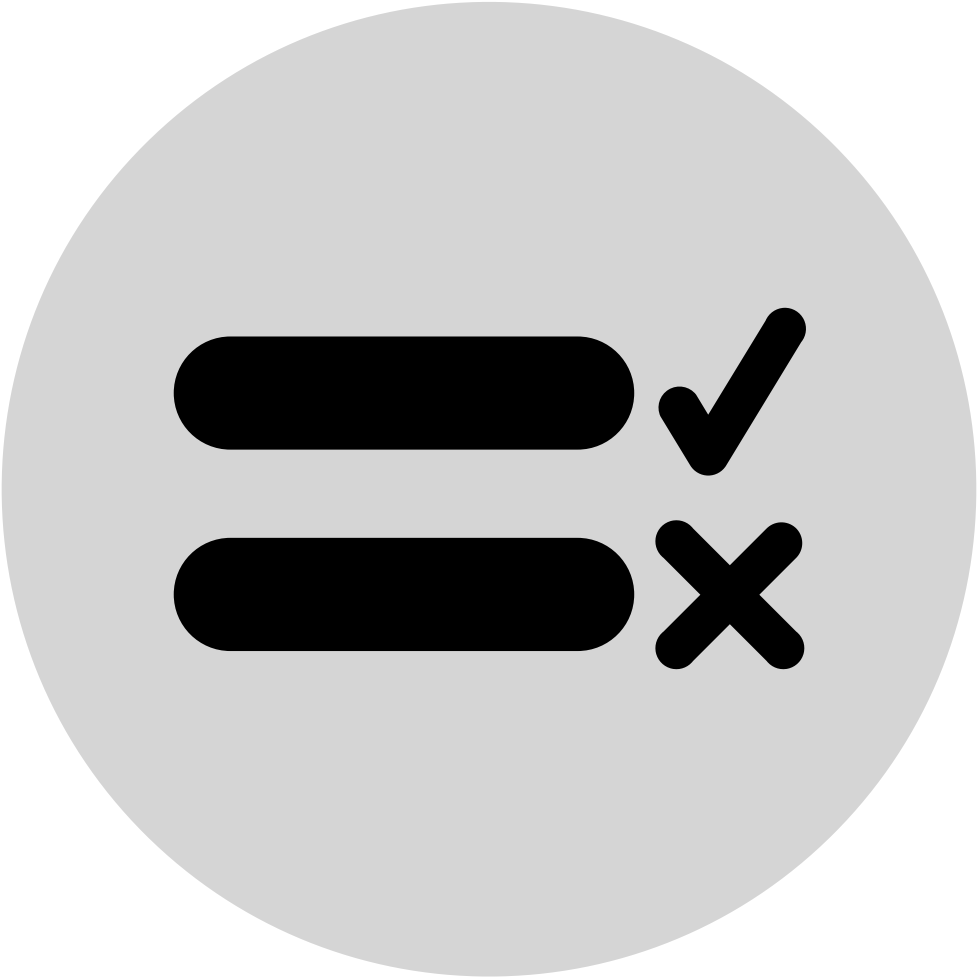


global principle

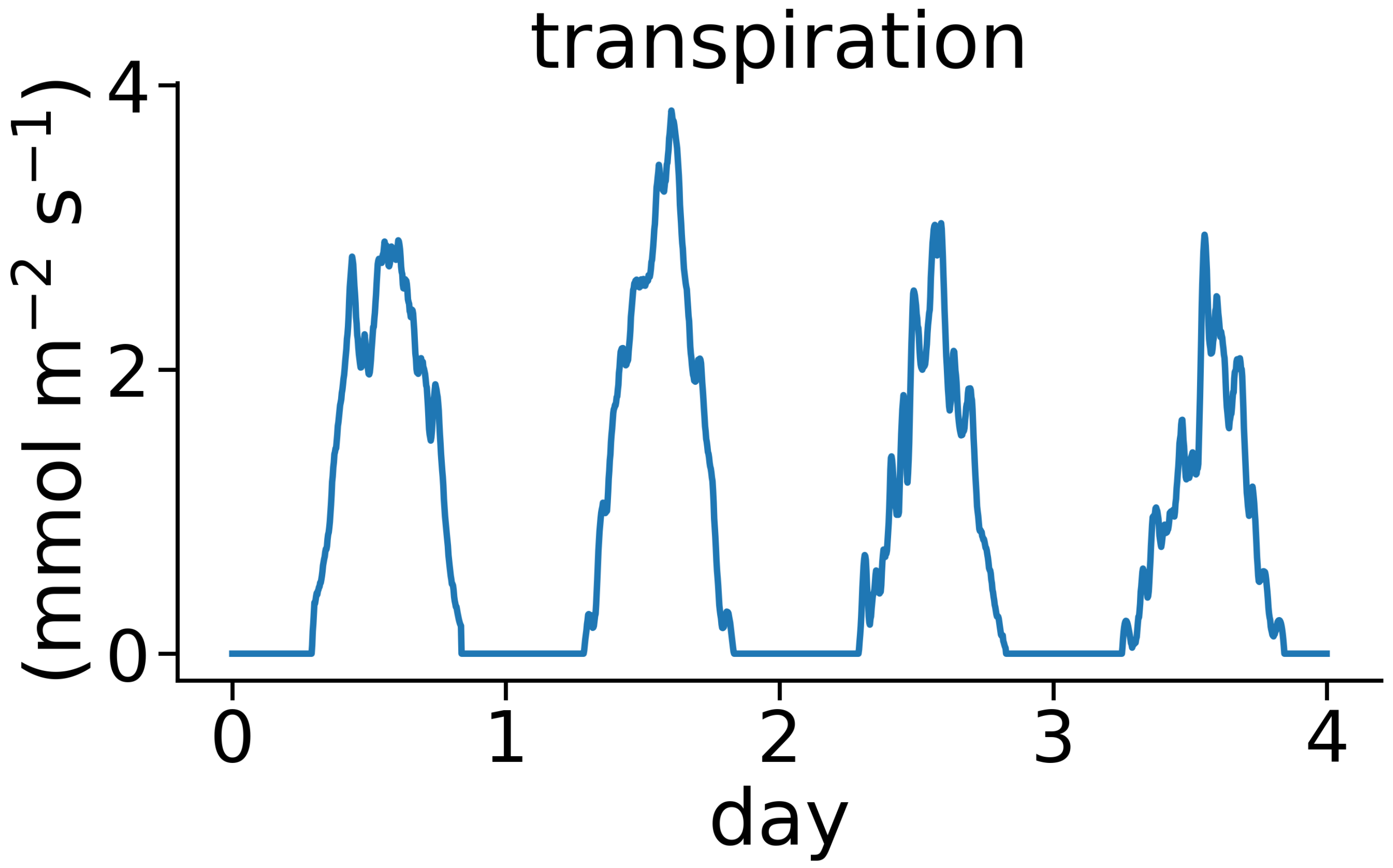
observed path

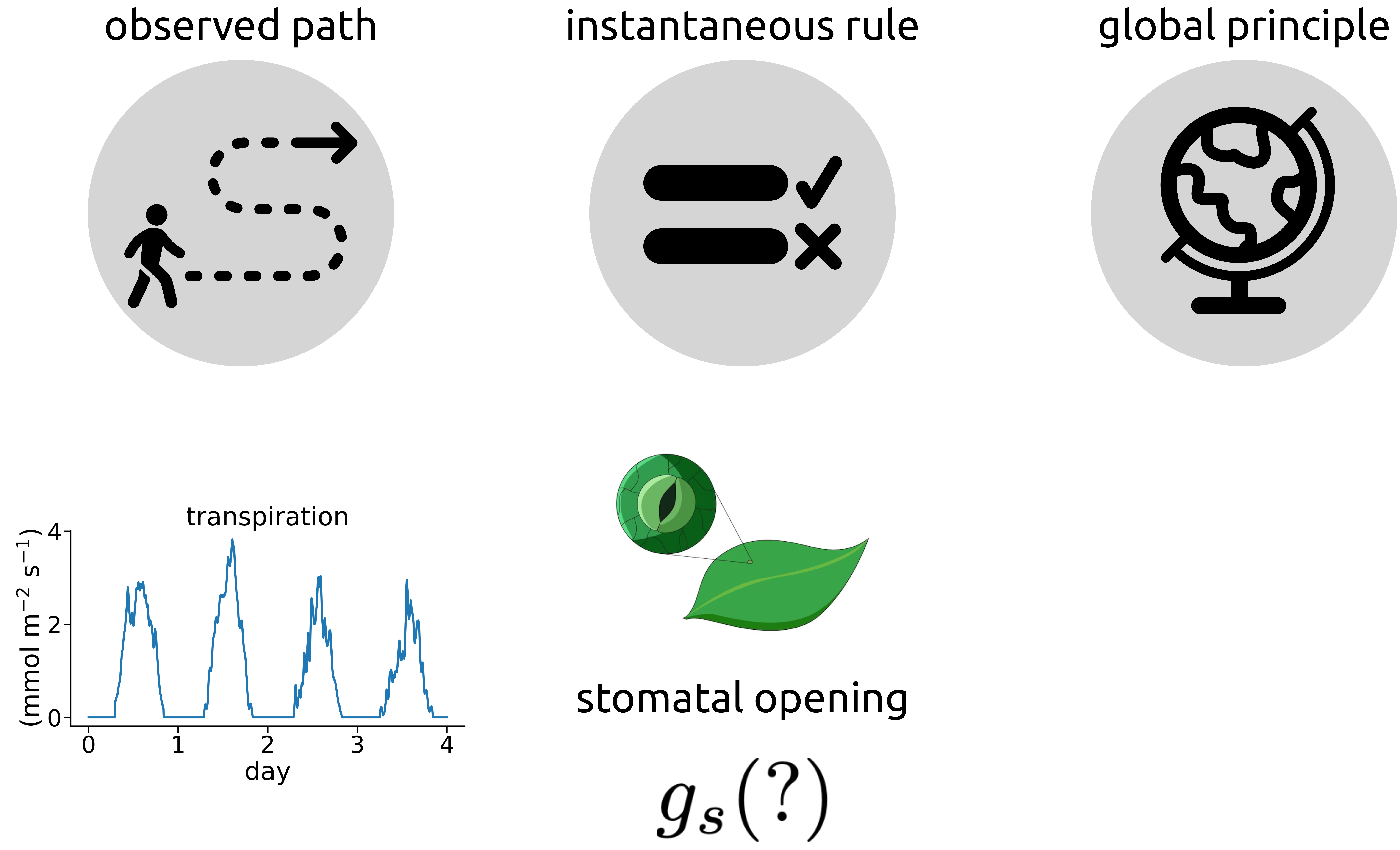


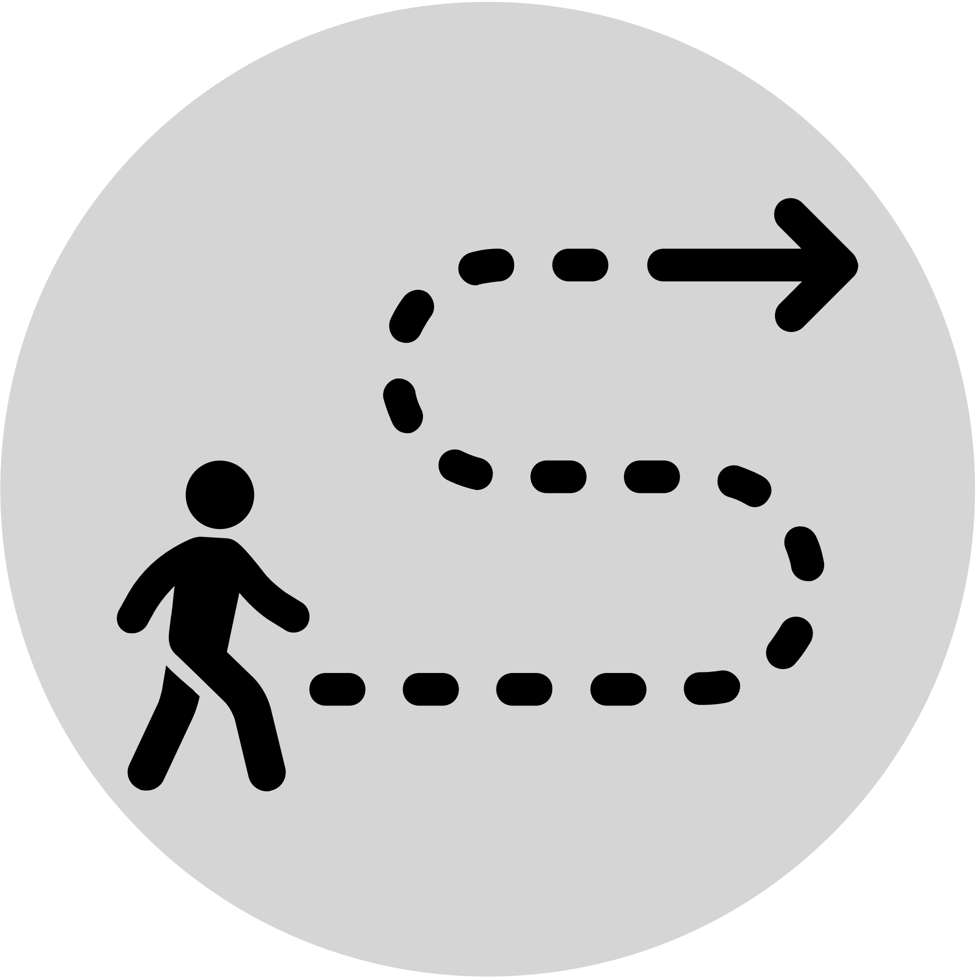
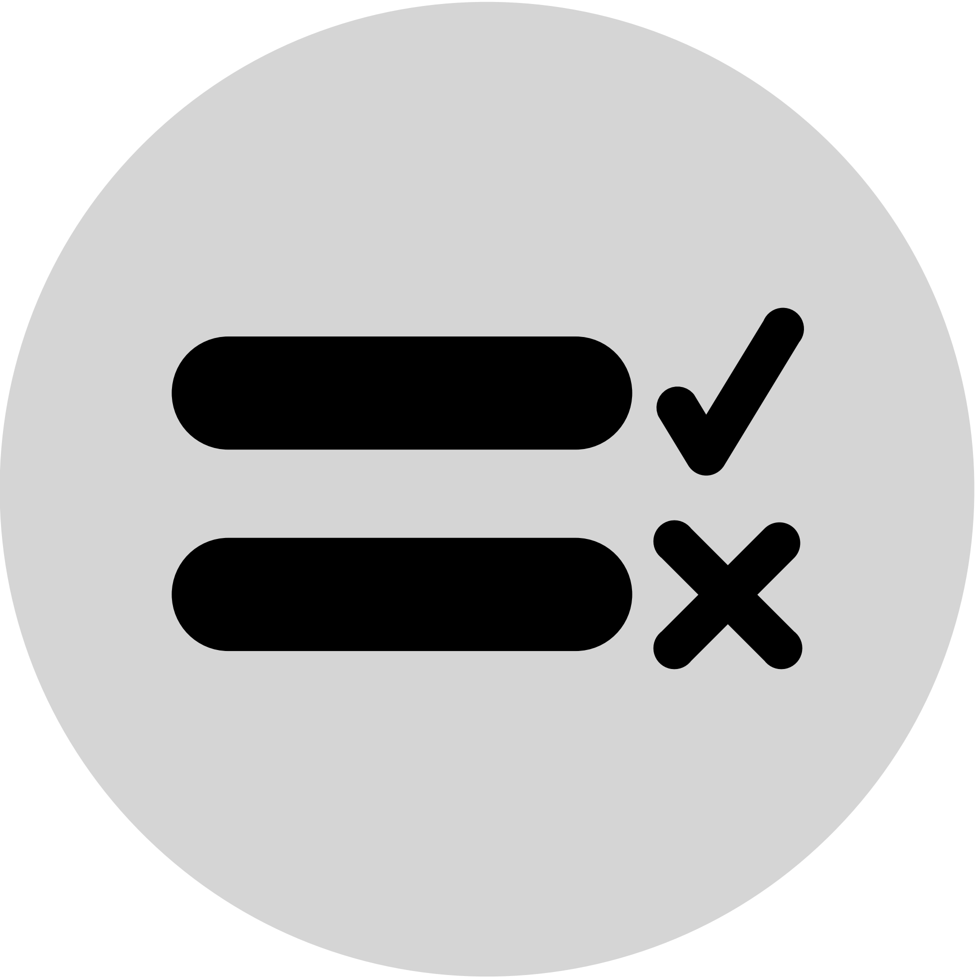
instantaneous rule

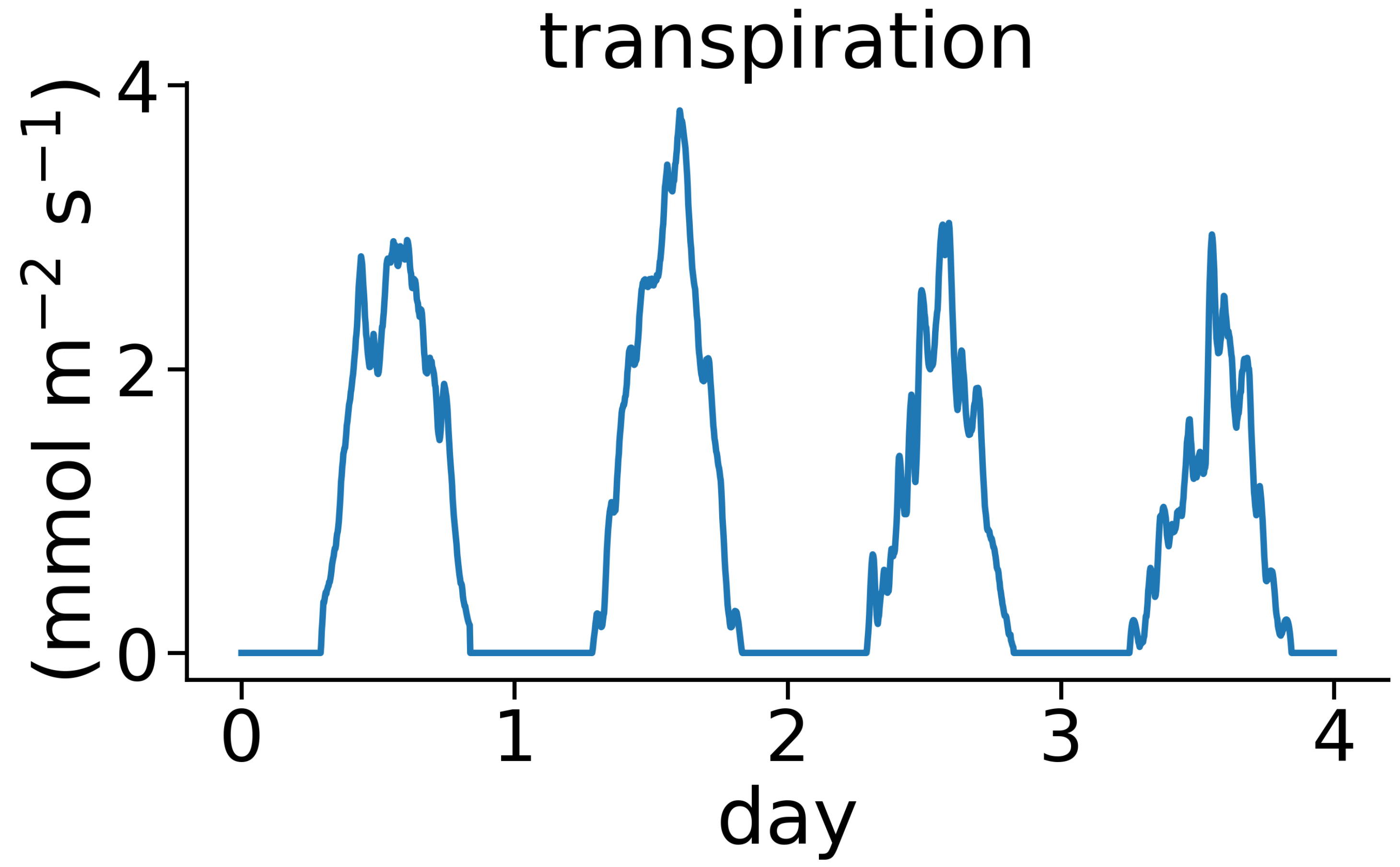


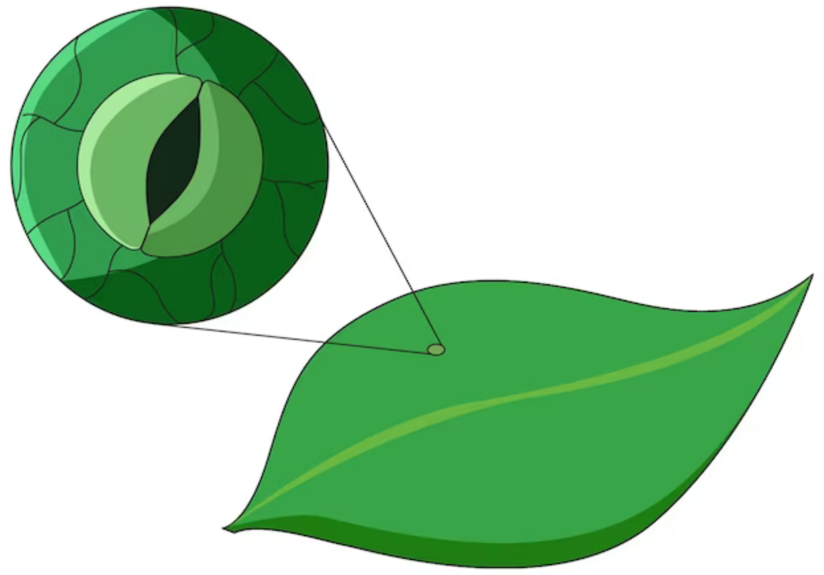
global principle




observed path
instantaneous rule
global principle
transpiration
(mmol m^{-2} s^{-1})
0
2
4
0
1
2
3
4
day
stomatal opening
$g_s(?)$

observed path	instantaneous rule	global principle
		

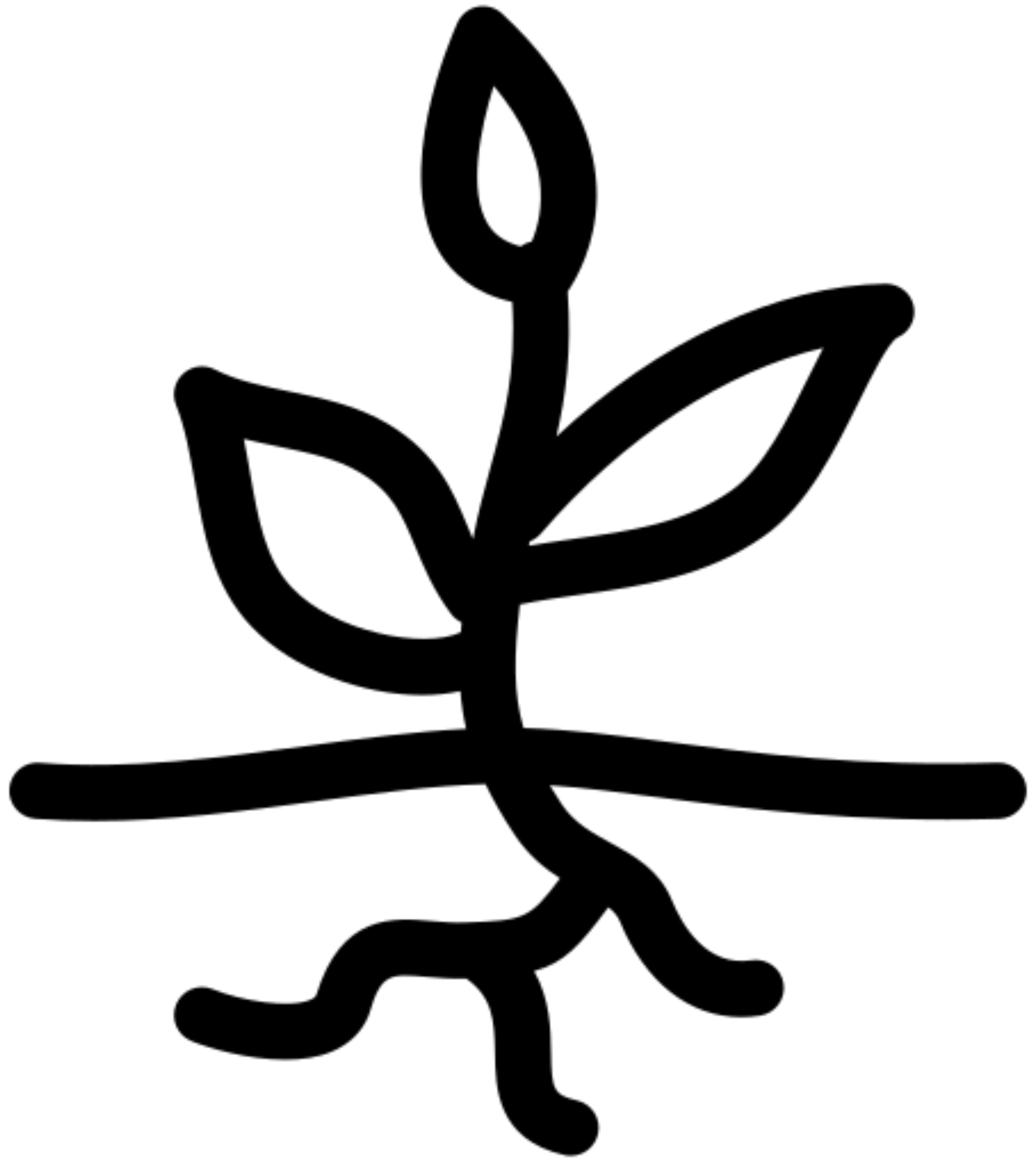




stomatal opening

$$g_s(?)$$

- How do plants respond to drought stress?
- What are plants optimizing for?
- What are the most important traits that explain the plant's behavior?
- How do different plant species differ in their water management strategies?




SUPER
MARIOKART™
PUSH B


3	acceleration	1
3	top speed	4
3.5	weight	5


2
1
3

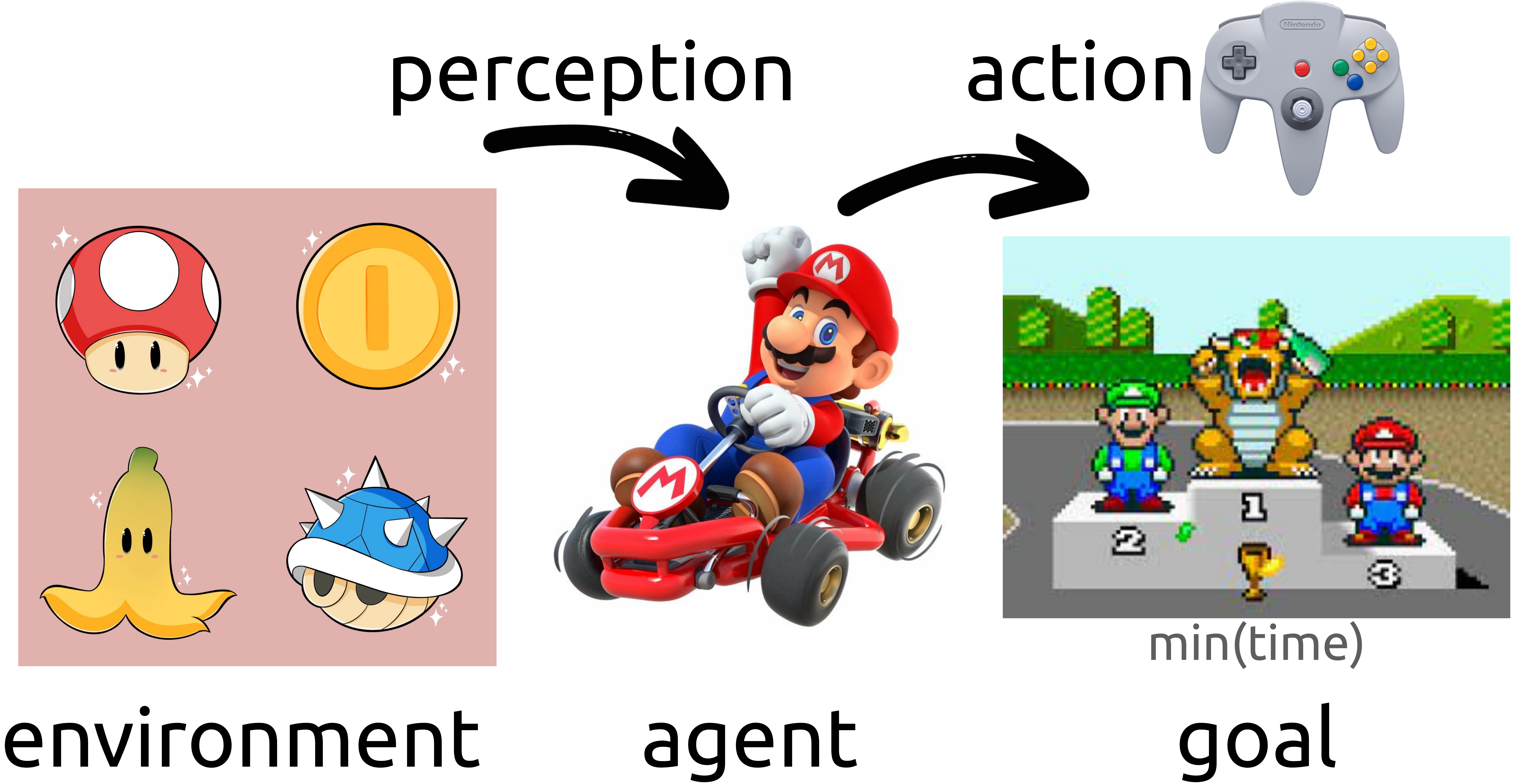
- intelligent agent
- perceives its environment
- takes actions autonomously
- in order to achieve goals
- may improve its performance with learning or may use knowledge

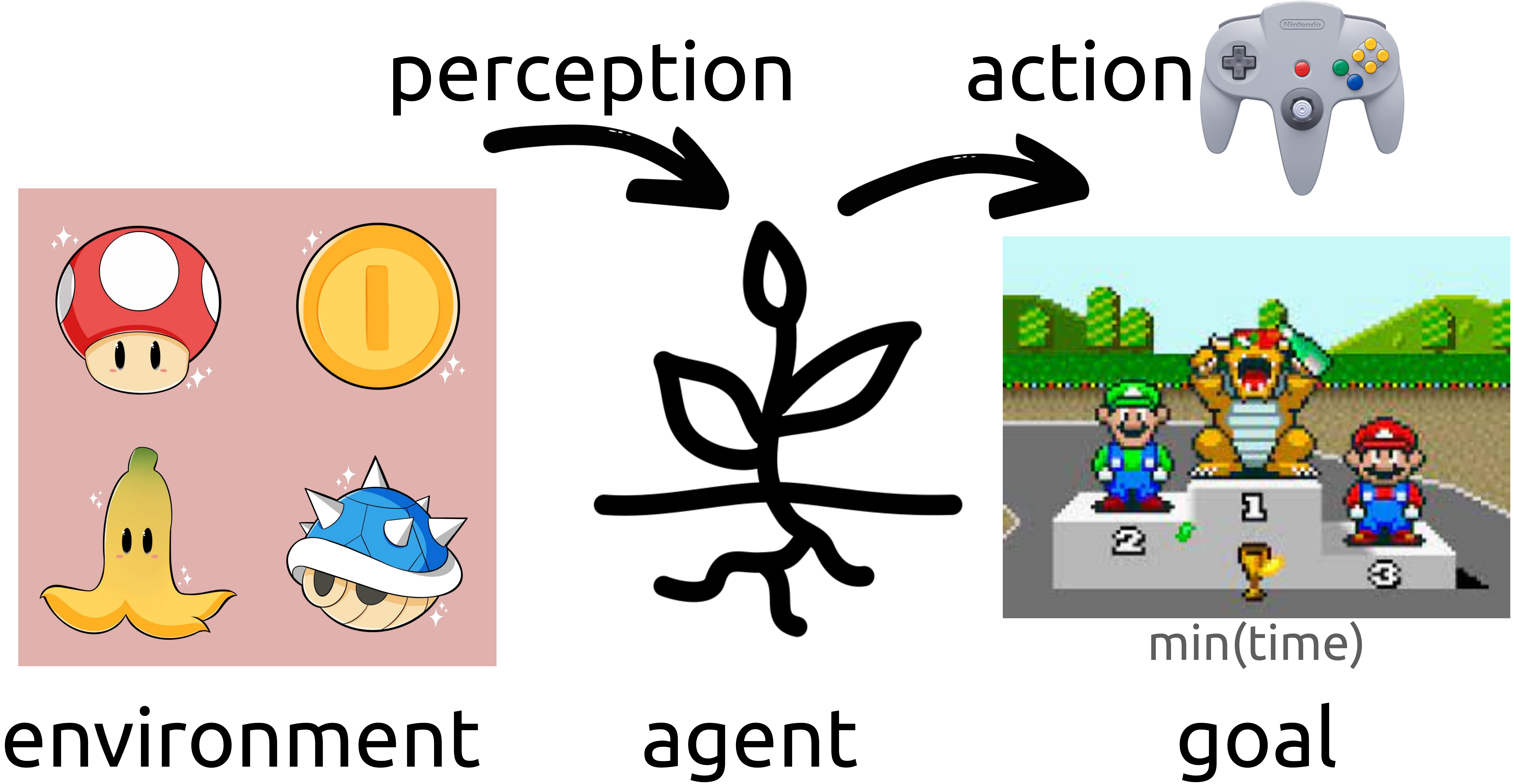
agent

perception

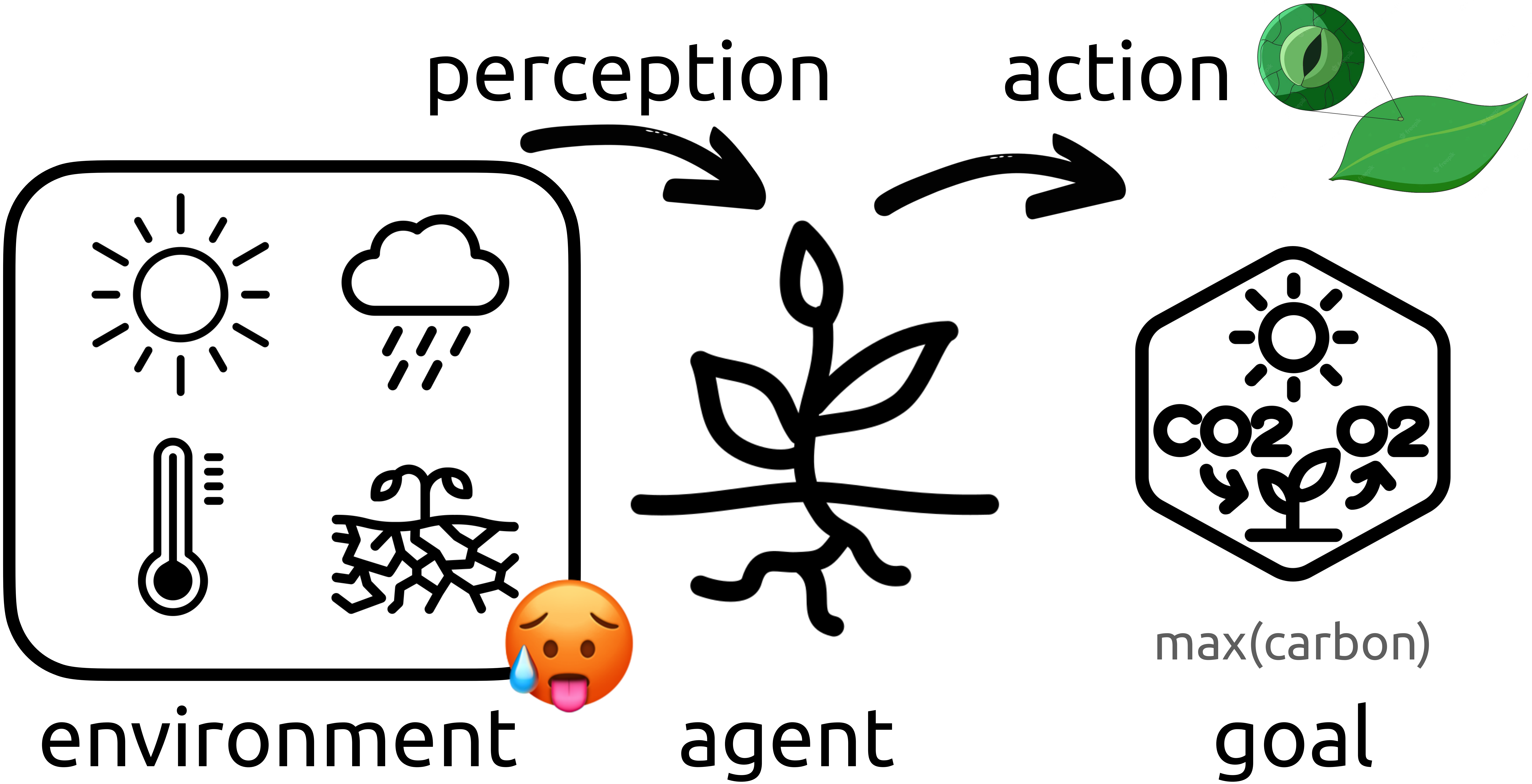
environment

agent

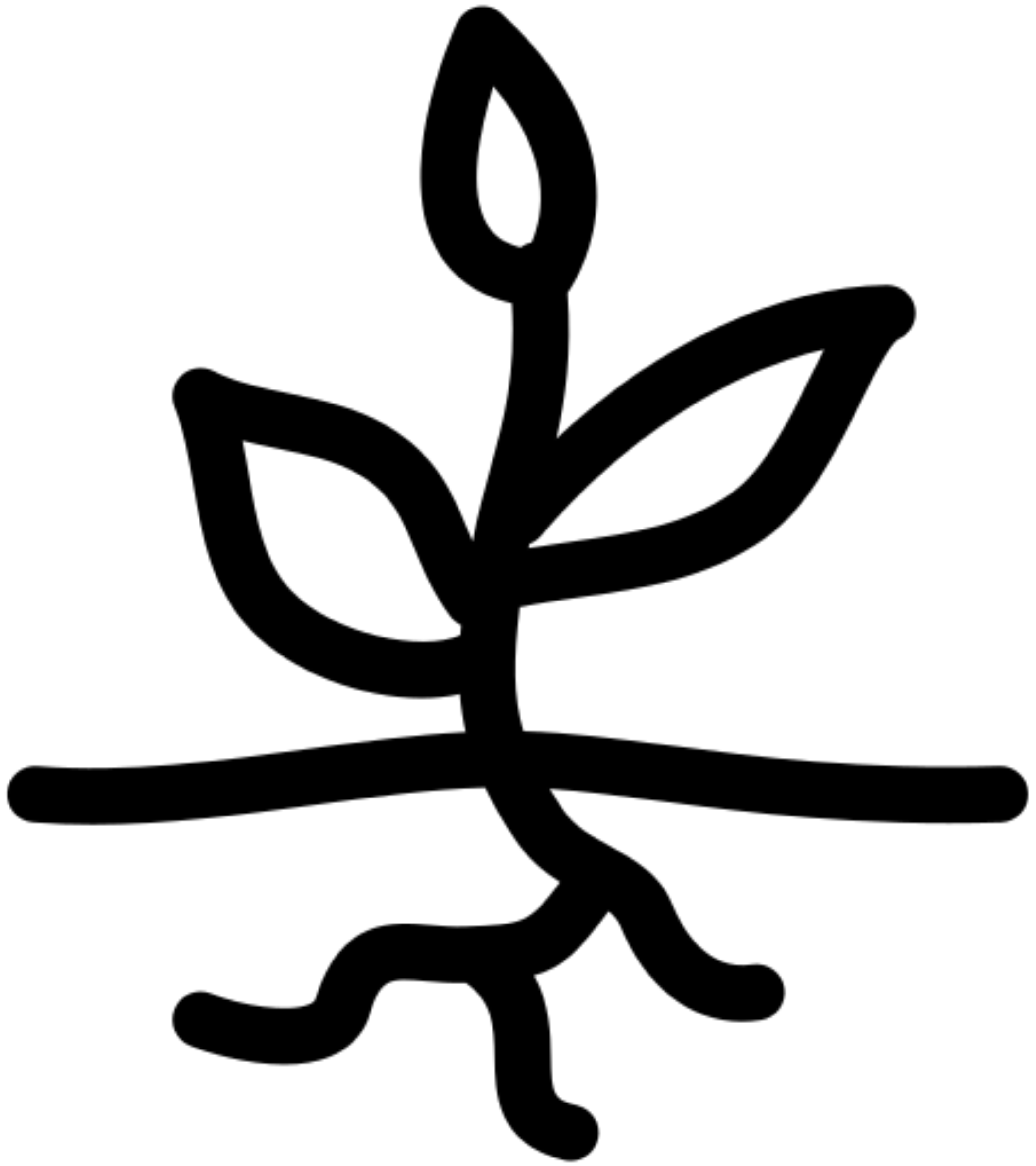

perception
action
min(time)
environment
agent
goal


perception
action
min(time)
environment
agent
goal


perception
action
min(time)
environment
agent
goal


perception
action
CO2
O2
max(carbon)
environment
agent
goal

- intelligent agent
- perceives its environment
- takes actions autonomously
- in order to achieve goals
- may improve its performance with learning or may use knowledge



keywords: artificial intelligence, machine learning, reinforcement learning, optimal control theory

- intelligent agent
- perceives its environment
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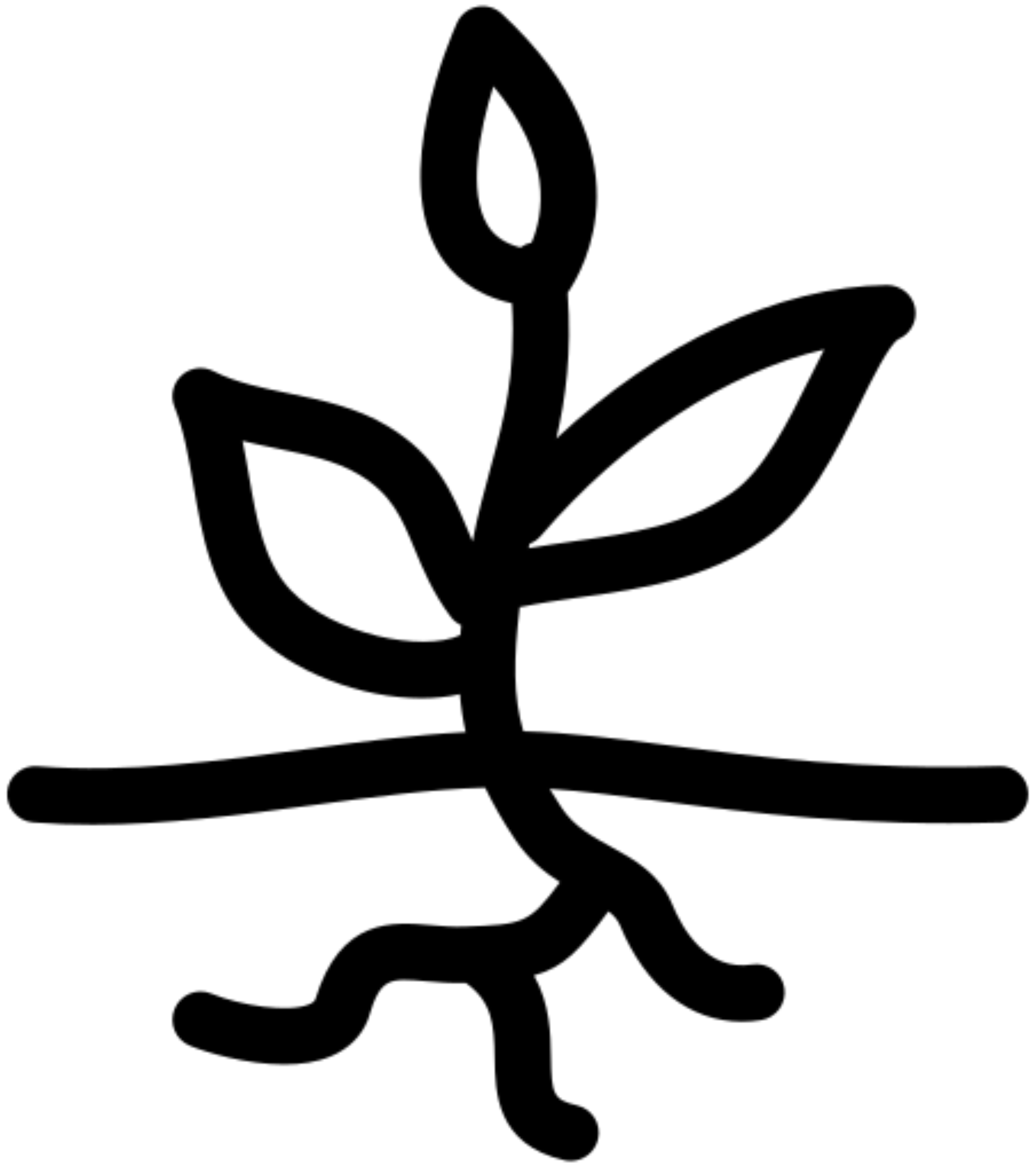

what I care about

3	**water use efficiency**	1
3	**resilience to drought**	4
3.5	**risk policy**	5

strategy 1: drive at full throttle

- there's only here and now
- tomorrow? who cares

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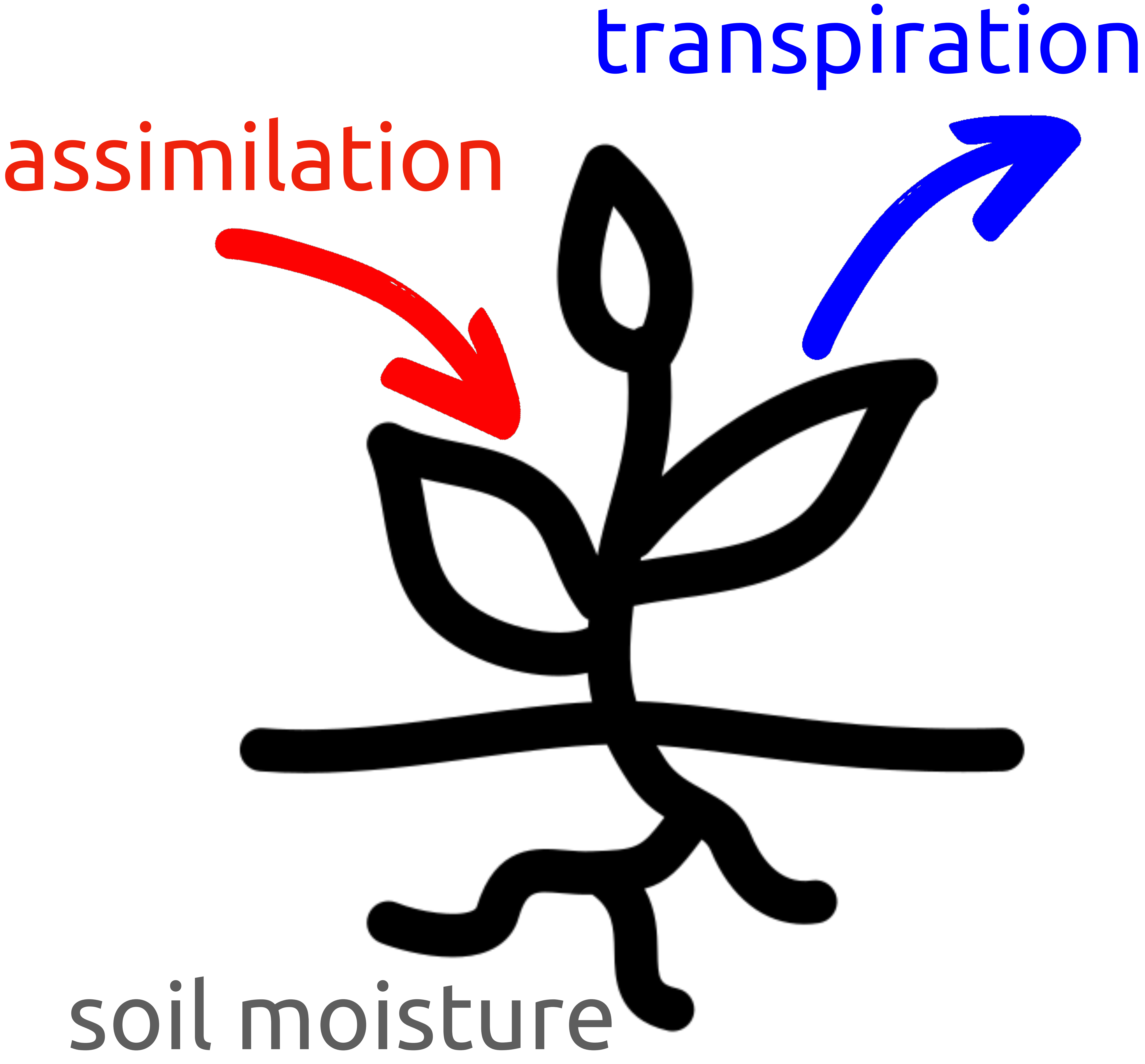
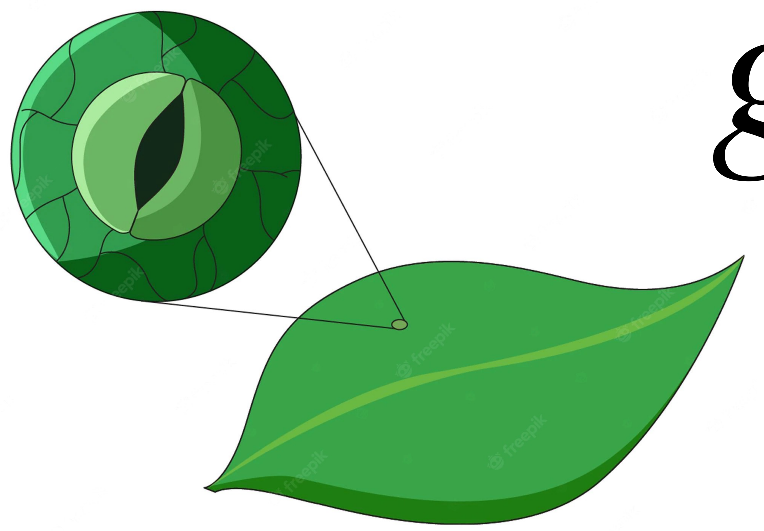


strategy 1: drive at full throttle

instantaneously optimize

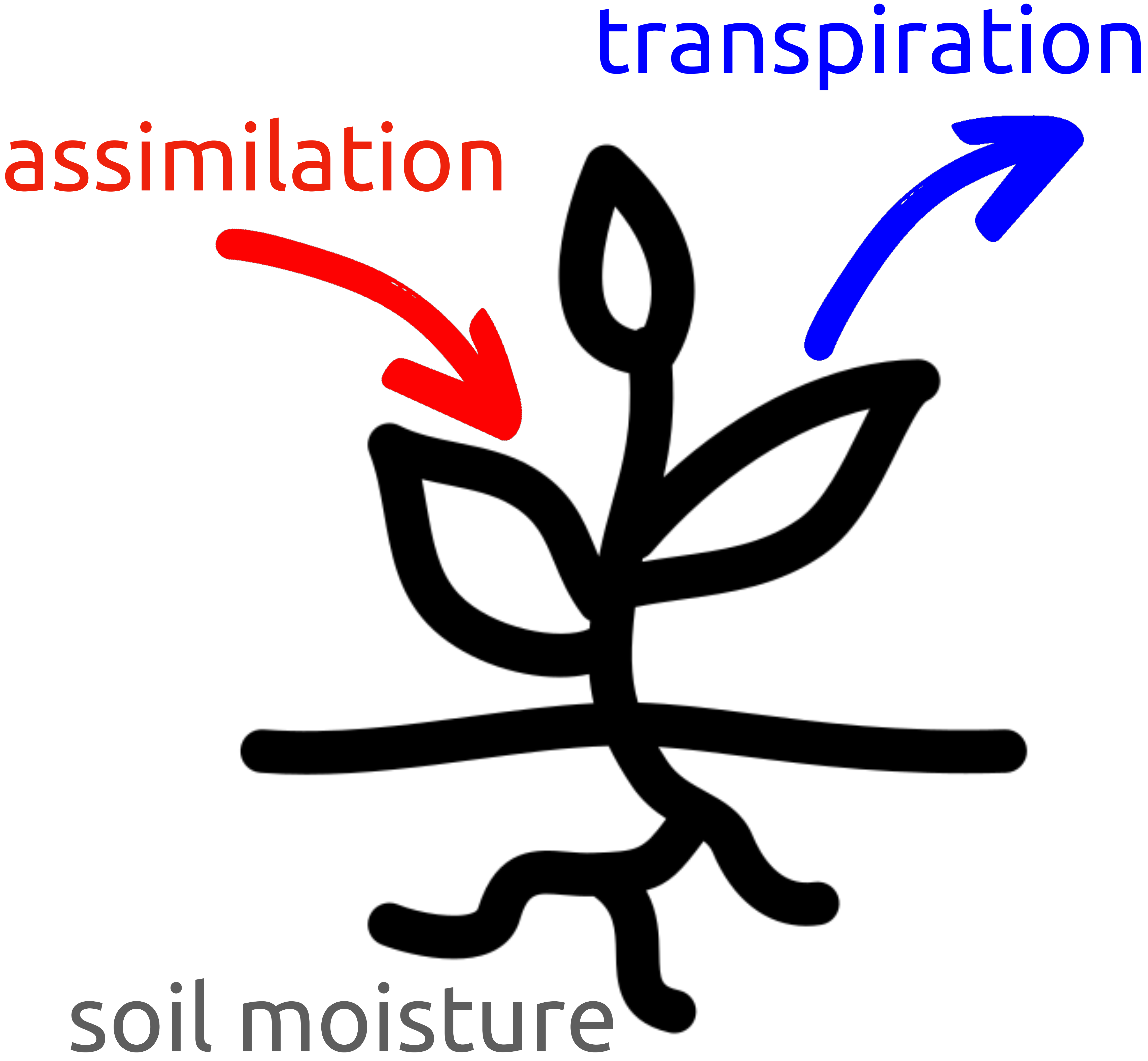
$$H = A(g_s) - \lambda \cdot E(g_s)$$

$$\lambda = \frac{\partial A}{\partial E}$$ water use efficiency



$g_s(t)$ is such that

H is maximum



tomato: 12-day drydown

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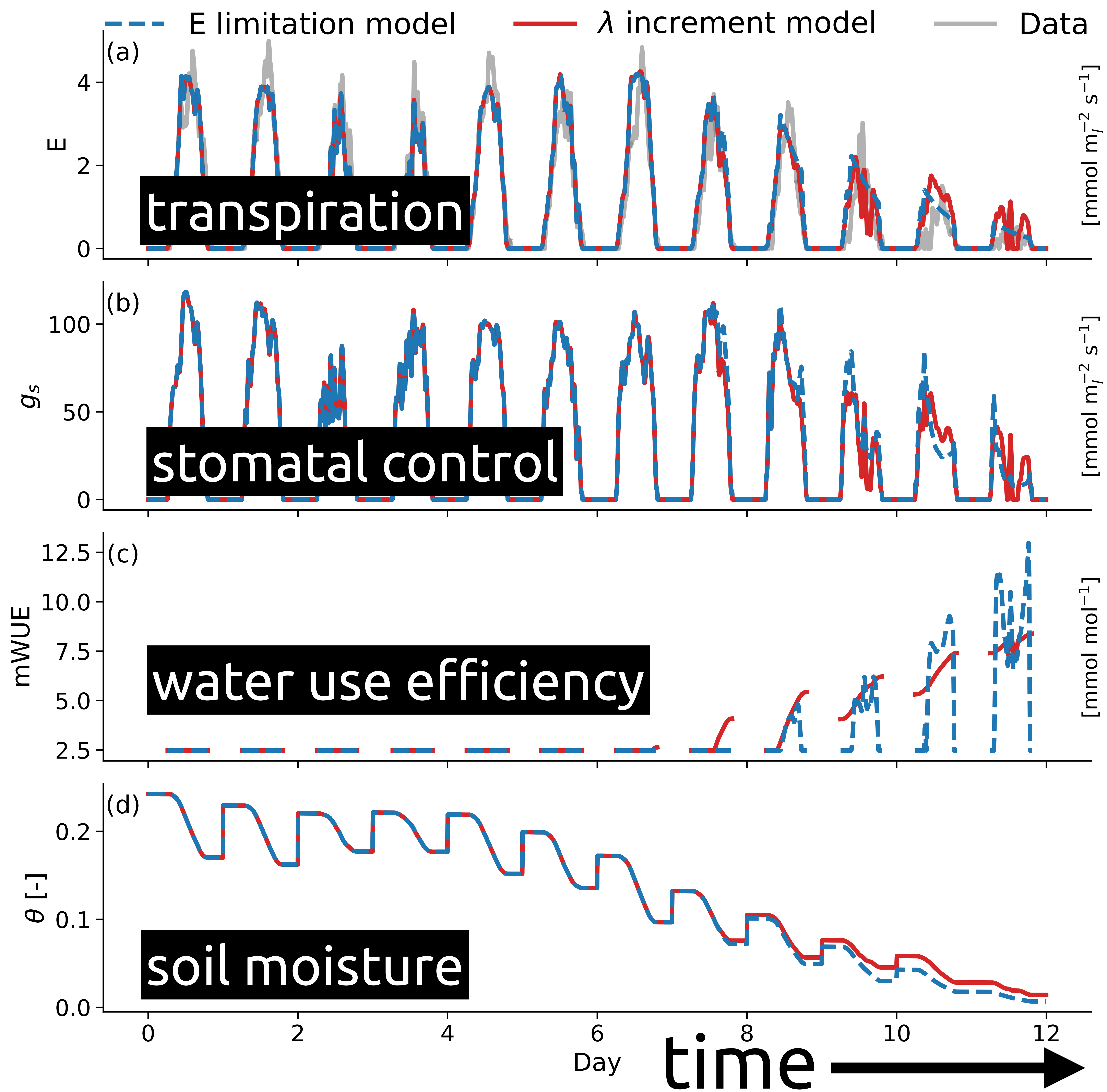
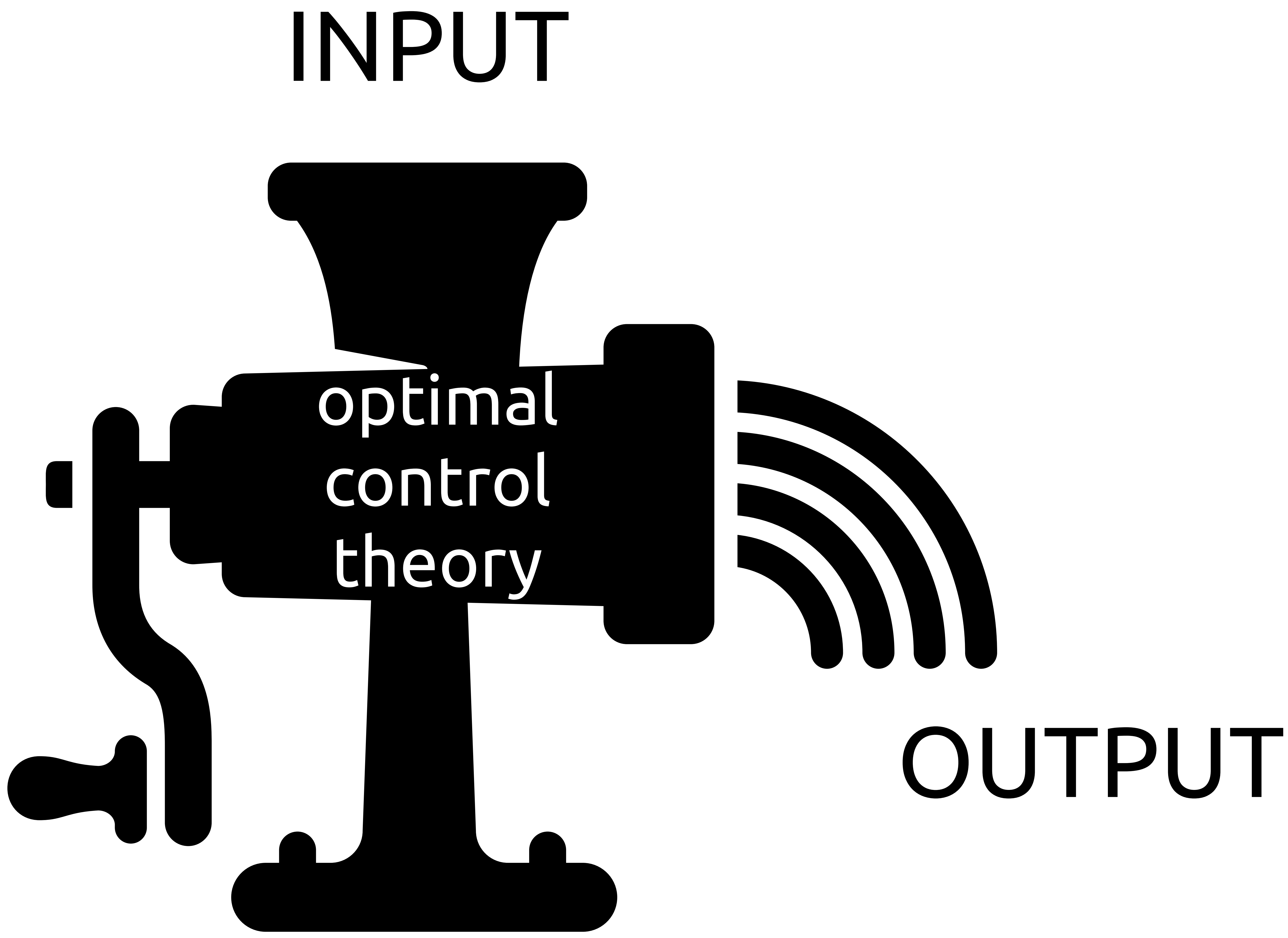


tomato: 12-day drydown


INPUT
optimal
control
theory
OUTPUT

INPUT

maximize carbon assimilation

conservation of water
soil water → transpiration

$0 < g_s < g_s^{\mathrm{max}}$
g_s^{max} is f(soil water)

INPUT

maximize carbon assimilation

conservation of water

soil water → transpiration

$0 < g_s < g_s^{\text{max}}$

g_s^{max} is f(soil water)

OUTPUT

g_s(VPD, light, T, CO_2)

water use efficiency

vulnerability to drought

Result 1

👍 **validation**

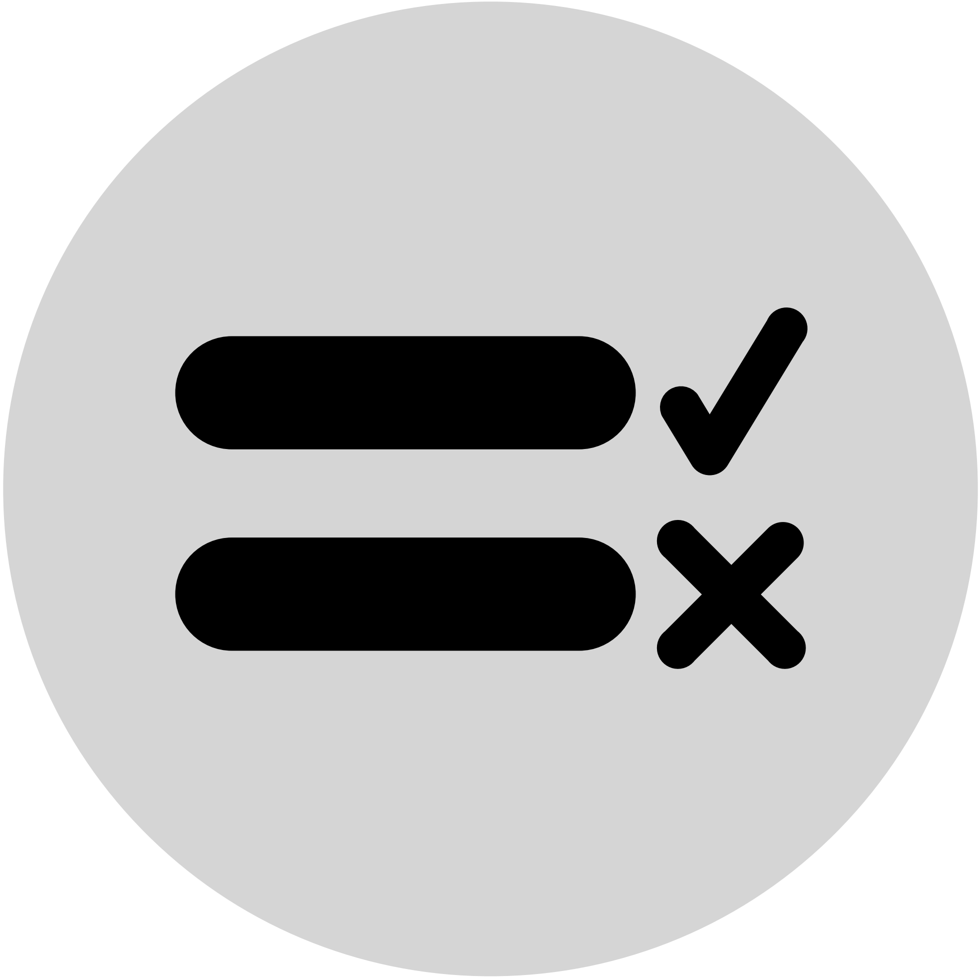
results are consistent with instantaneous optimization

Result 1

👍 **validation**

results are consistent with instantaneous optimization

instantaneous rule



$$\tilde{g}_s = \frac{k_1(C_a - k_2 - 2\Gamma^*)}{\beta^2} + (\beta - 2\alpha D\lambda)k_1 \frac{\sqrt{\alpha D\lambda(C_a - \Gamma^*)(k_2 + \Gamma^*)(\beta - \alpha D\lambda)}}{\alpha D\lambda\beta^2(\beta - \alpha D\lambda)}$$

Result 2

💪 plant traits

Result 2

💪 **plant traits**

water use efficiency

$$\lambda = \frac{\partial\,\text{assimilation}}{\partial\,\text{transpiration}}$$

Result 2

💪 plant traits

water use efficiency

$$\lambda = \frac{\partial\,\text{assimilation}}{\partial\,\text{transpiration}}$$

vulnerability to dry soil

$$E_{\max} = k \times \text{soil water}$$

Result 2

💪 plant traits

water use efficiency

$$\lambda = \frac{\partial \, \mathrm{assimilation}}{\partial \, \mathrm{transpiration}}$$

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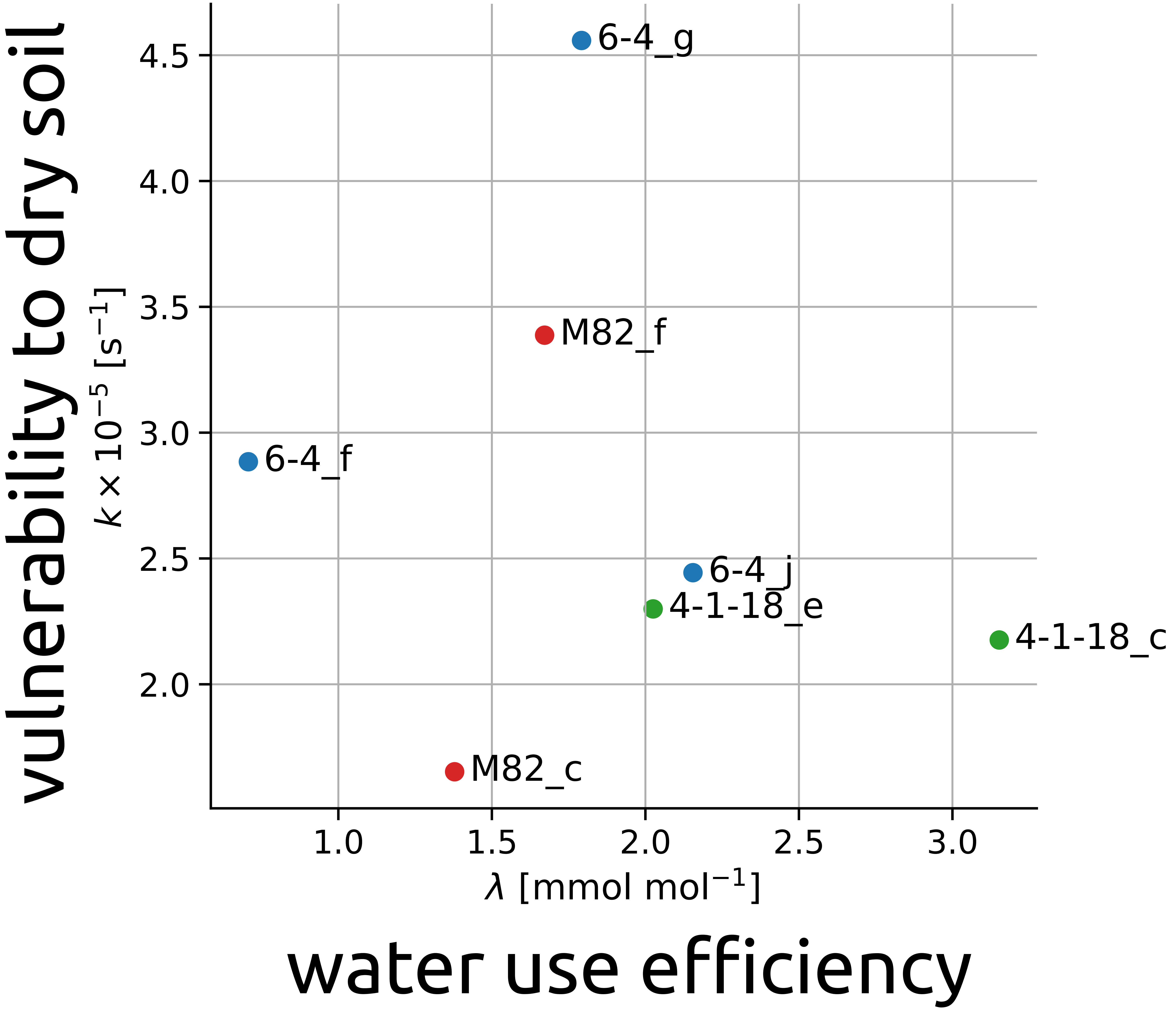


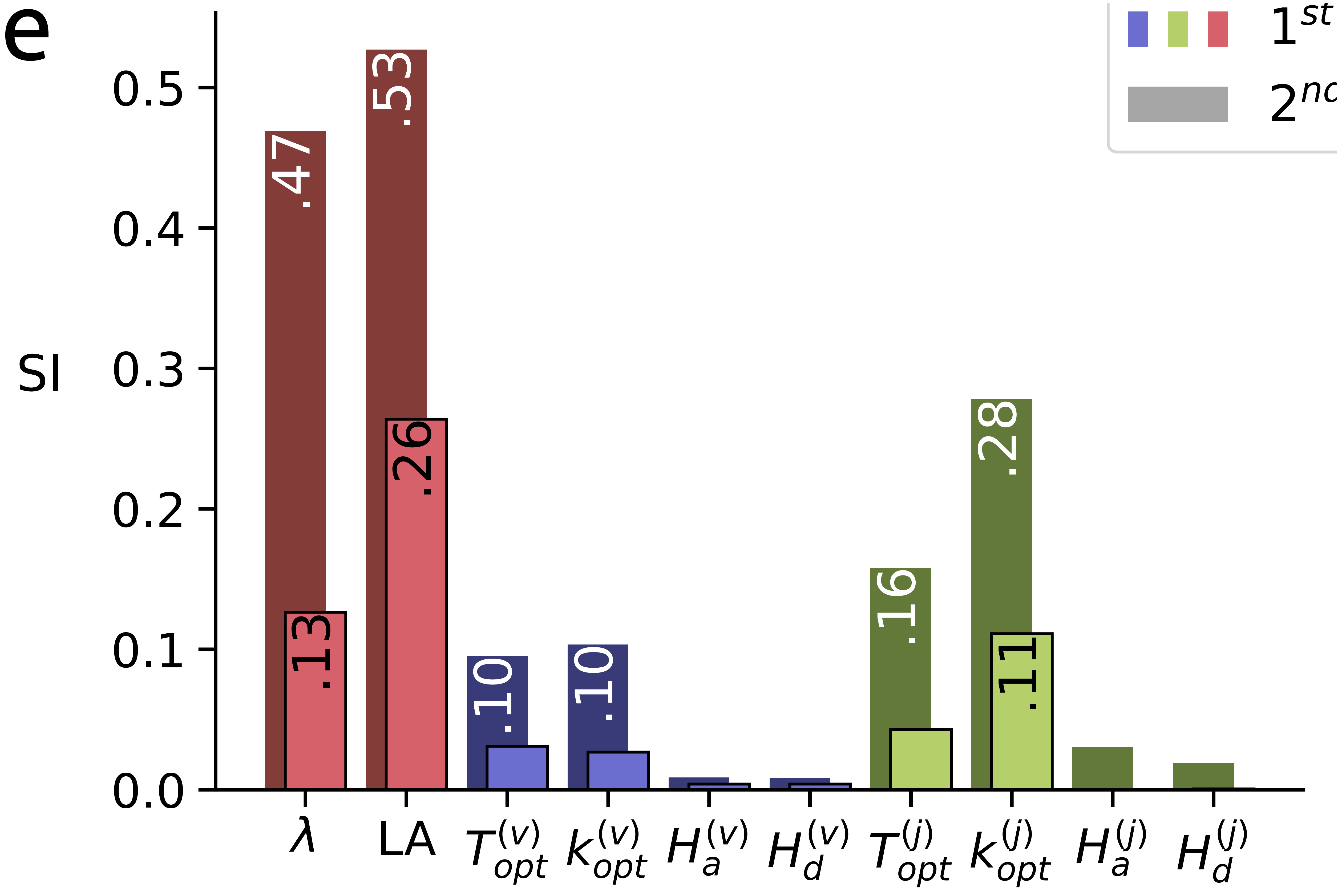
Result 3

😲 **(obvious) surprise**

(extensive parameters) pot size and leaf area

>

(intensive parameters) photosynthetic params.



strategy 2: beware of what’s ahead

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instantaneous maximization of $A(g_s)$ depleats soil moisture ***fast***

$$H = A(g_s) - \lambda \cdot E(g_s)$$

strategy 2: beware of what's ahead

instantaneous maximization of $A(g_s)$ depleats soil moisture ***fast***

$$H = A(g_s) - \lambda \cdot E(g_s)$$

plant should maximize $A(g_s)$ over time interval T

$$H = \frac{1}{T}\int_0^T A(g_s)dt - \lambda \cdot E(g_s)$$

strategy 2: beware of what's ahead

instantaneous maximization of $A(g_s)$ depleats soil moisture ***fast***

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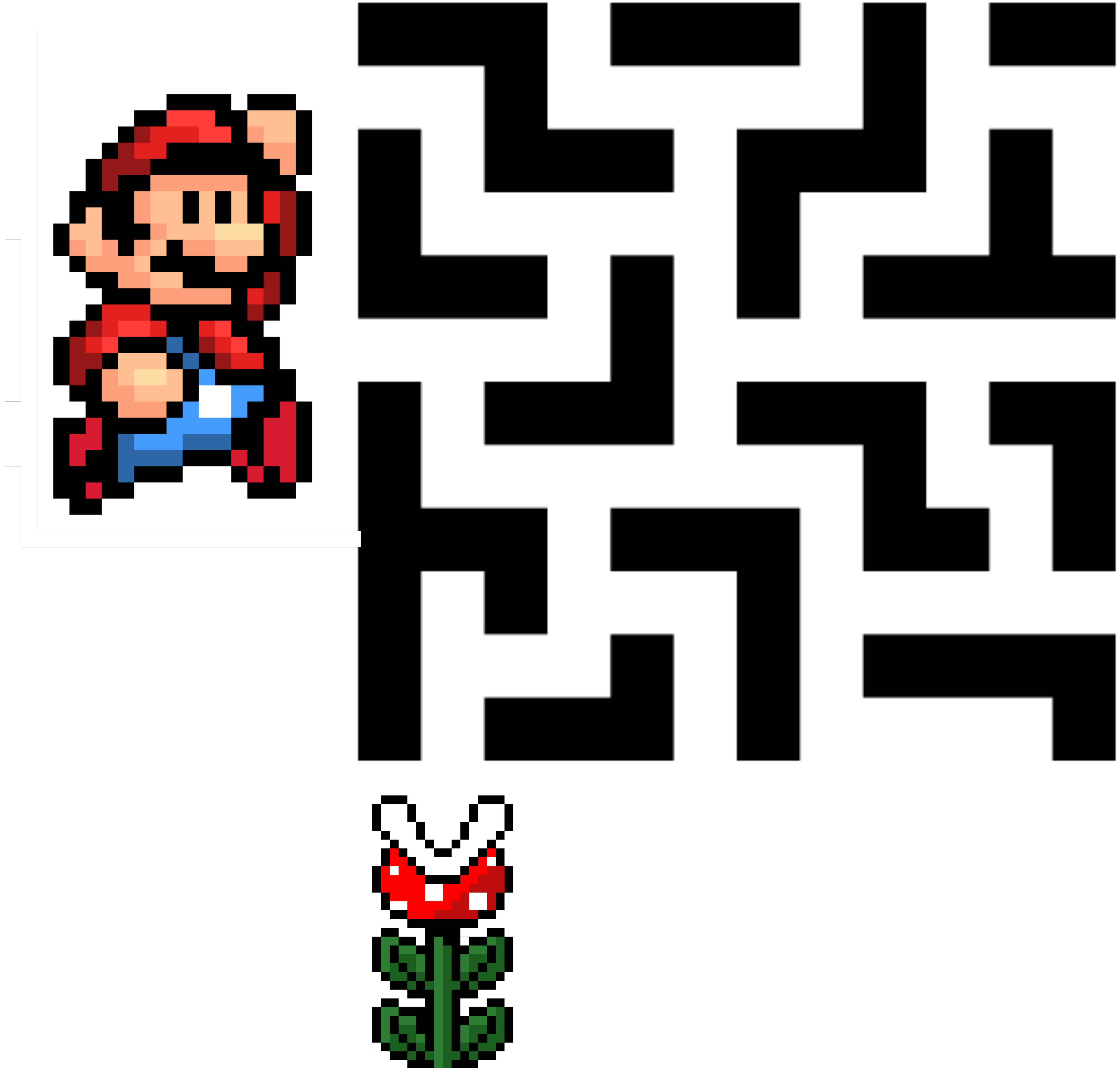
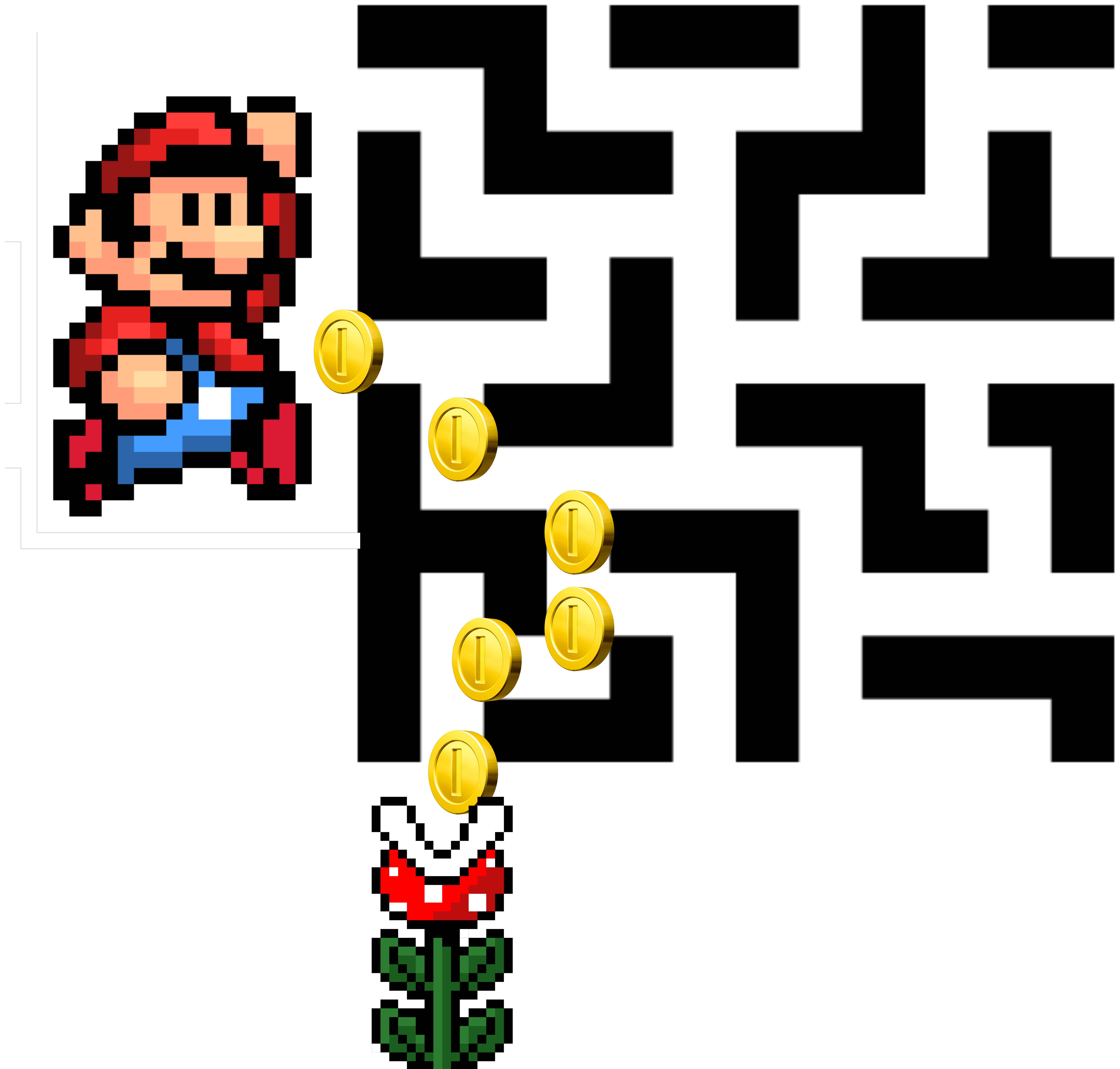
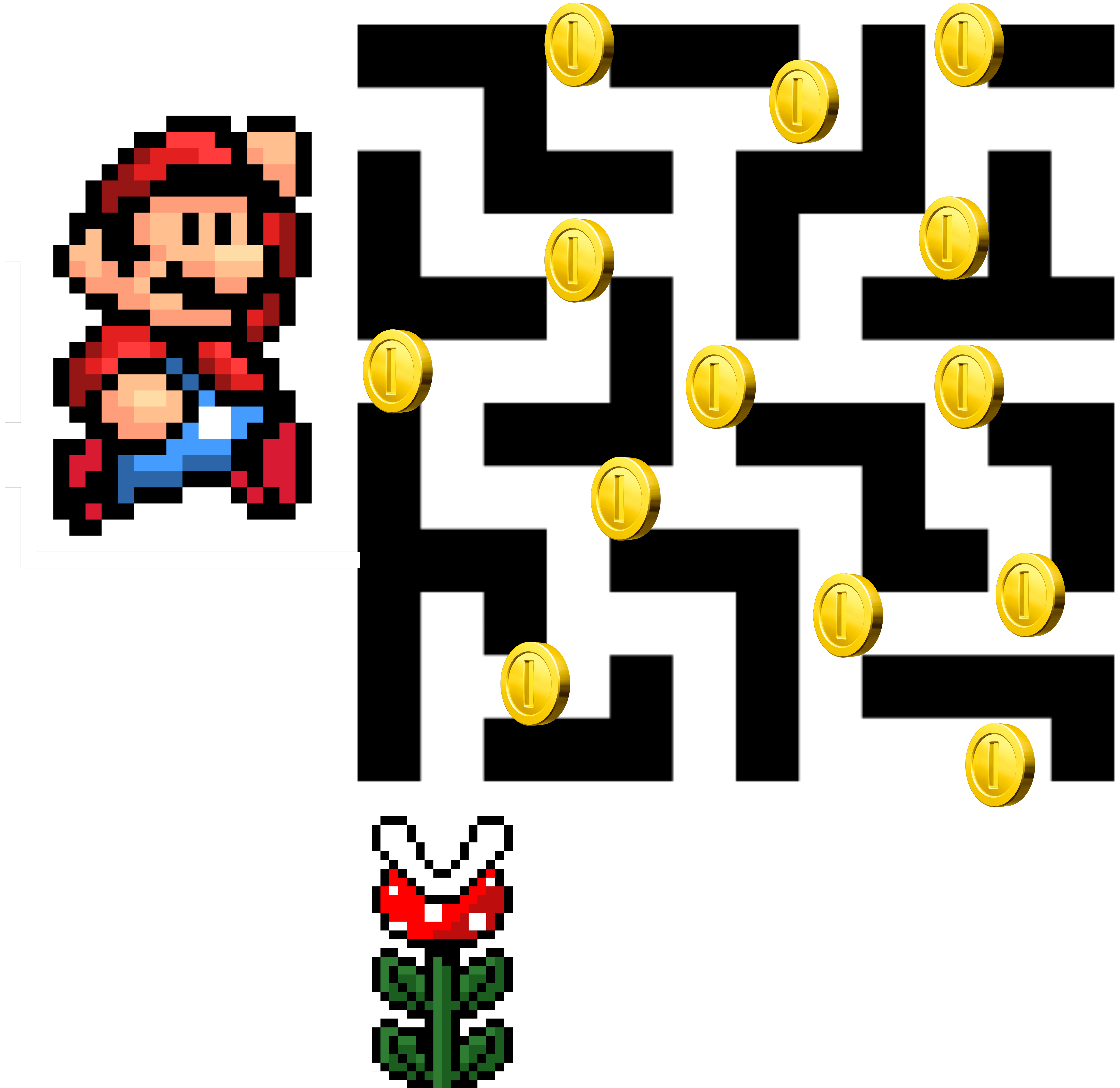
Cowan & Farquhar (1977), Mäkelä et al. (1996), Manzoni et al. (2013), Mrad et al. (2019)

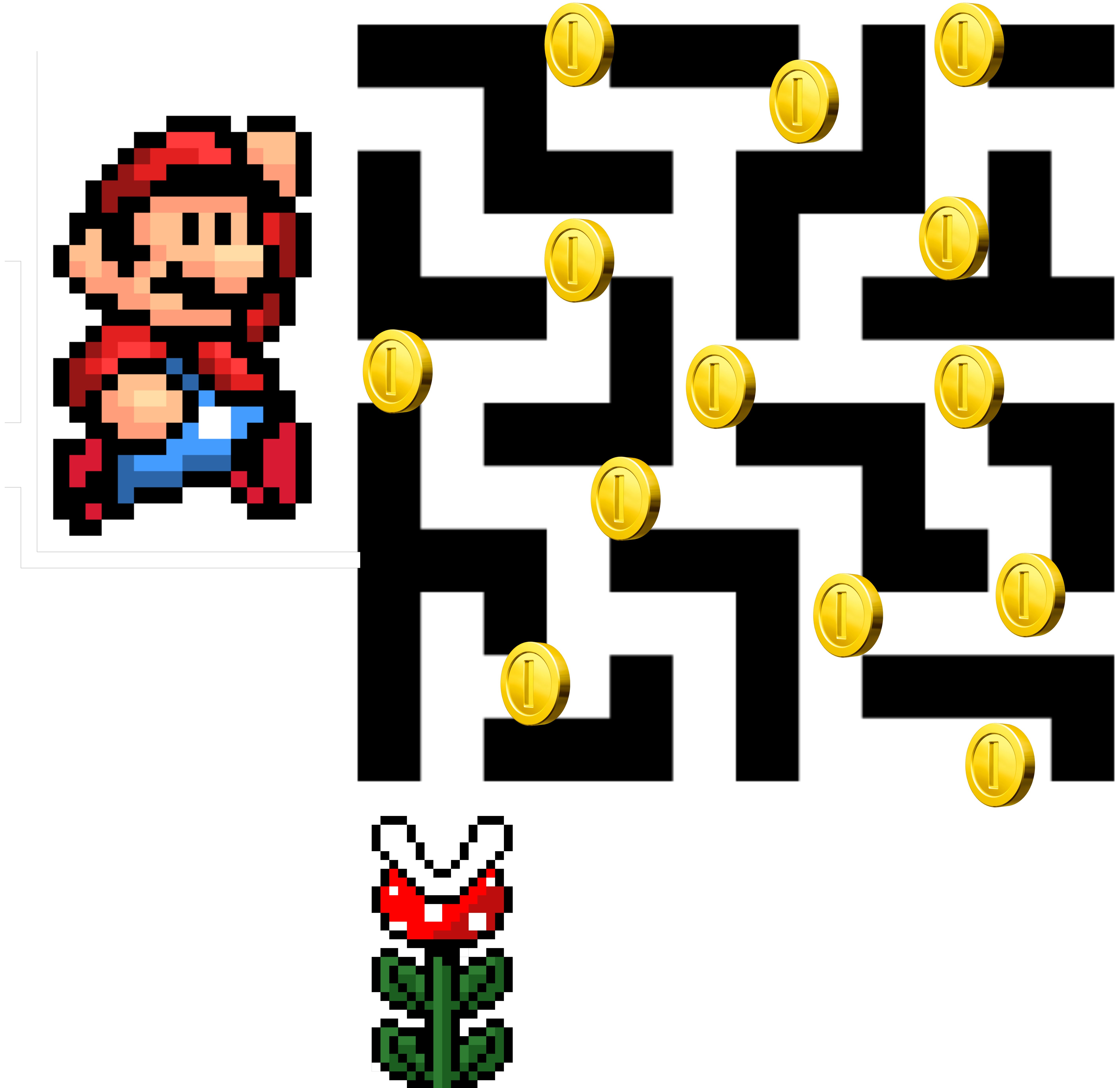




instantaneous maximization

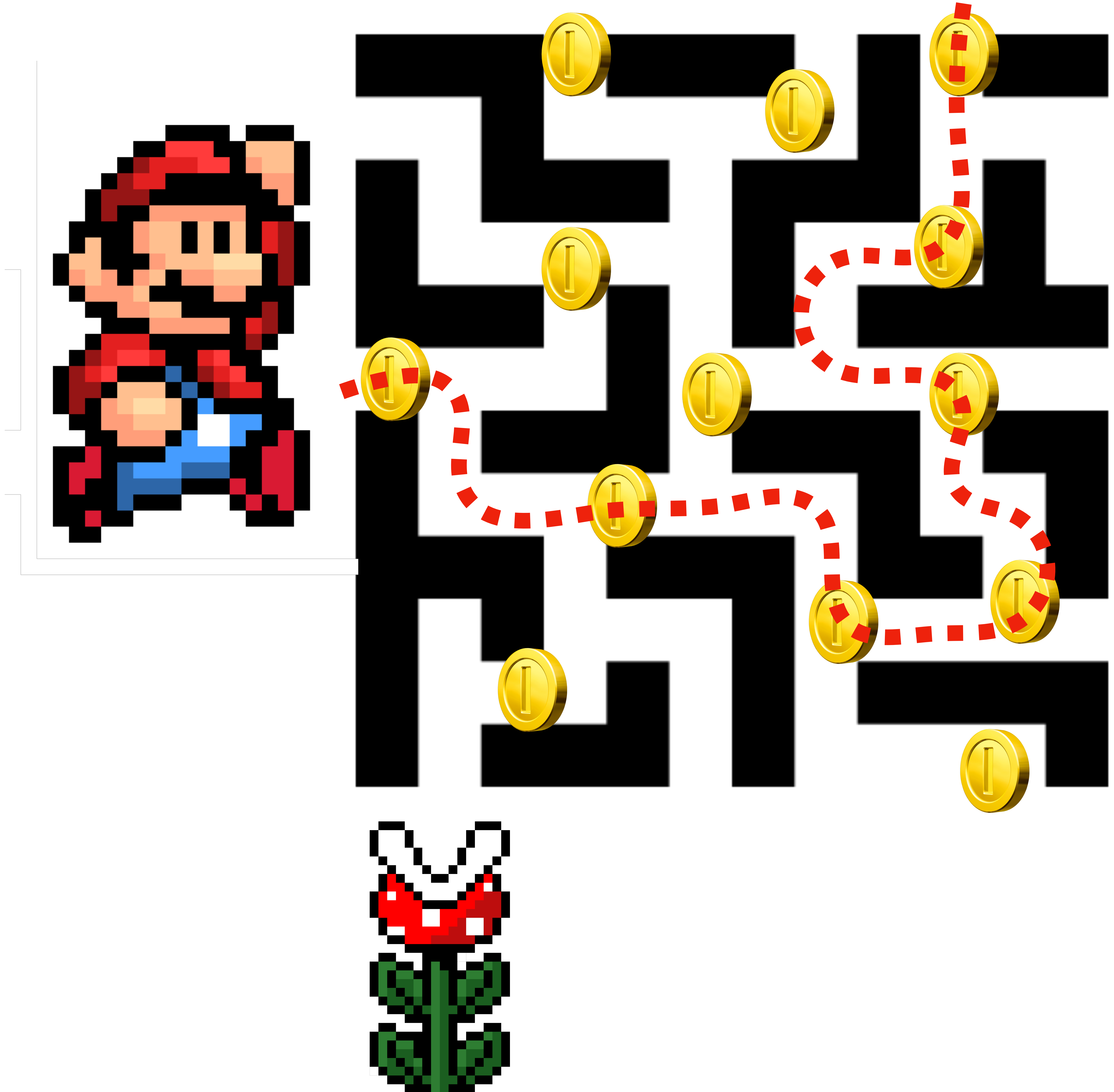
$A(g_s)$





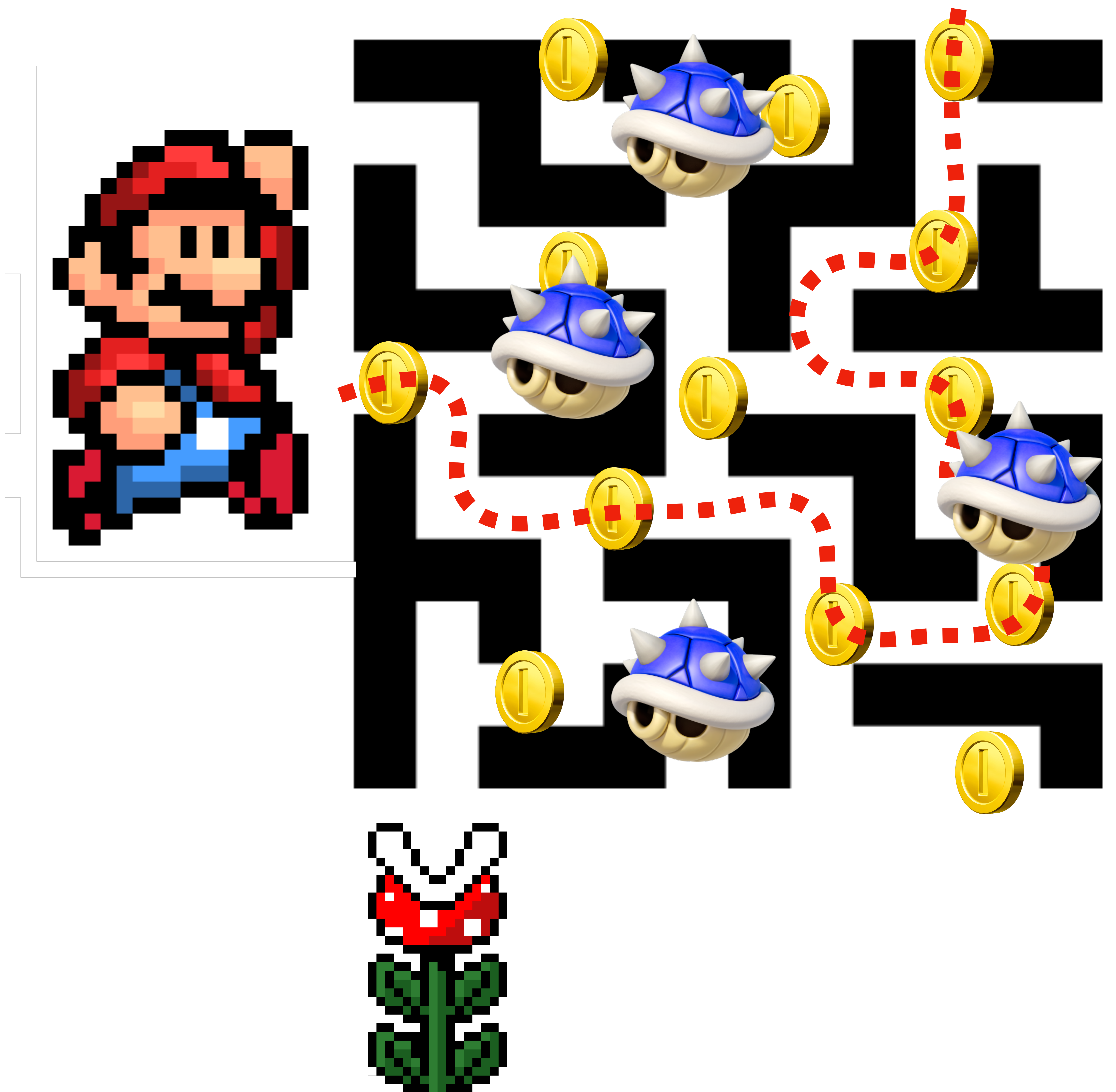
time integral

$$\frac{1}{T}\int_0^T A(g_s)dt$$



time integral

$$\frac{1}{T}\int_0^T A(g_s)dt$$



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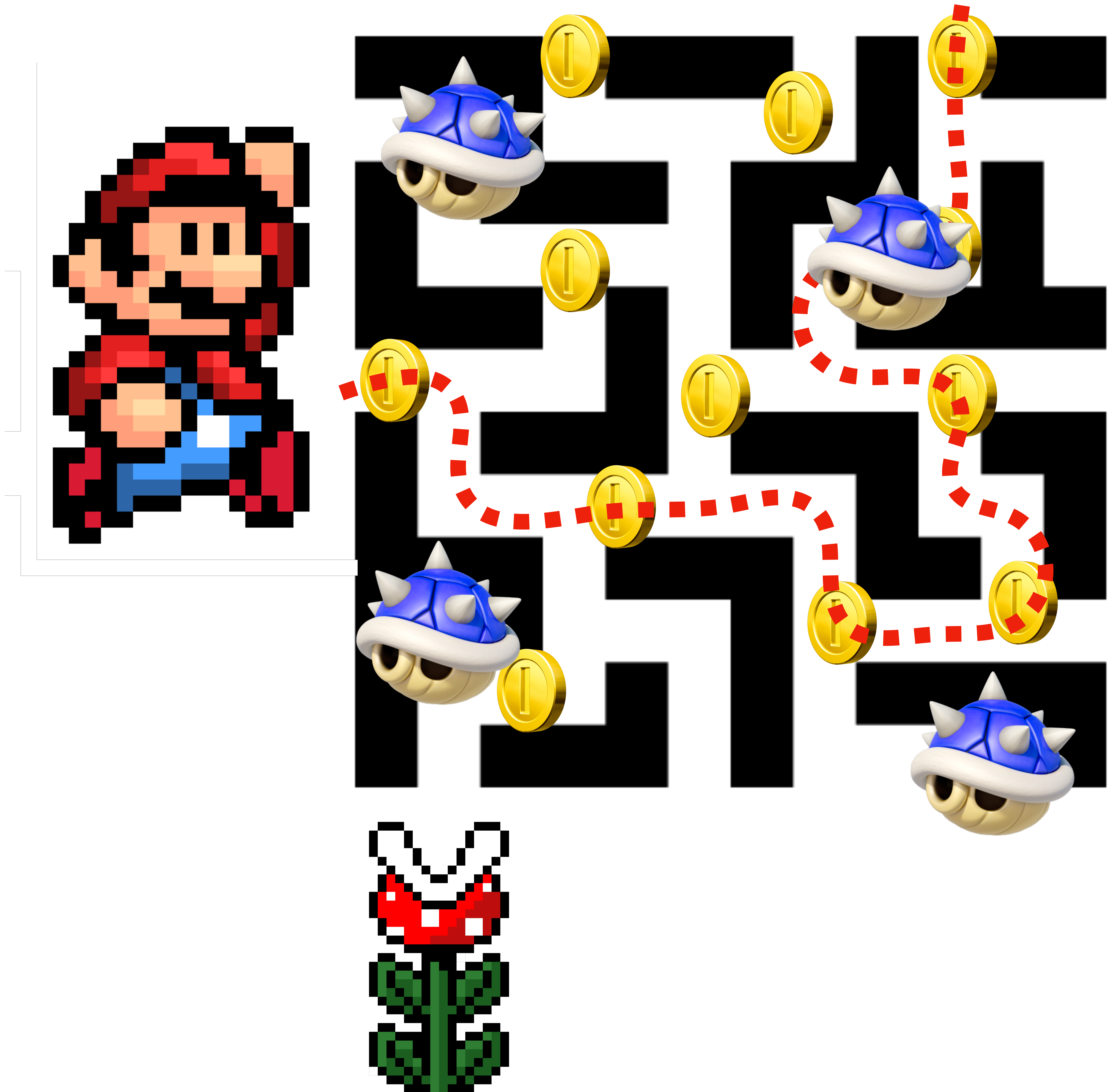

time integral

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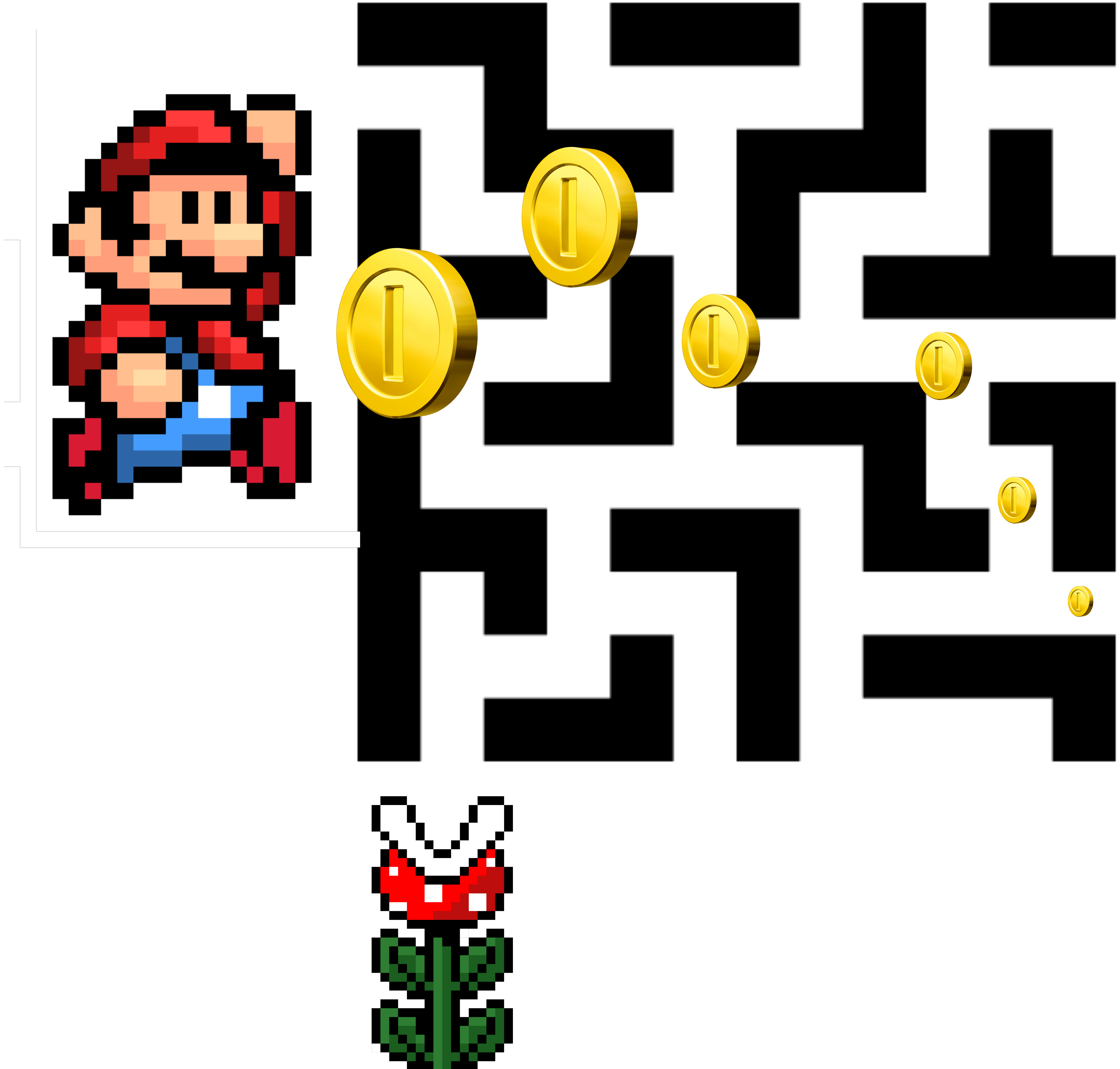
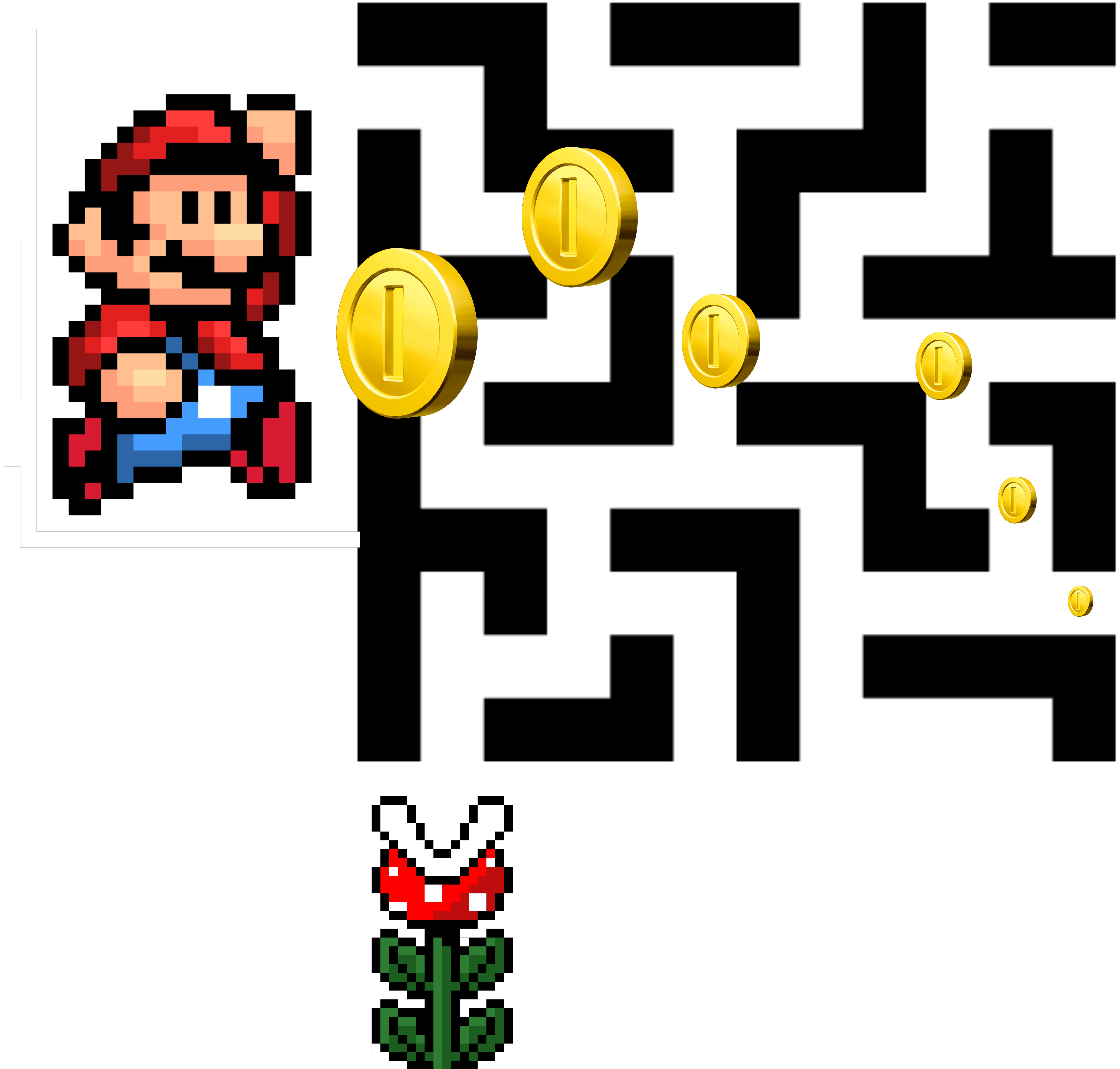
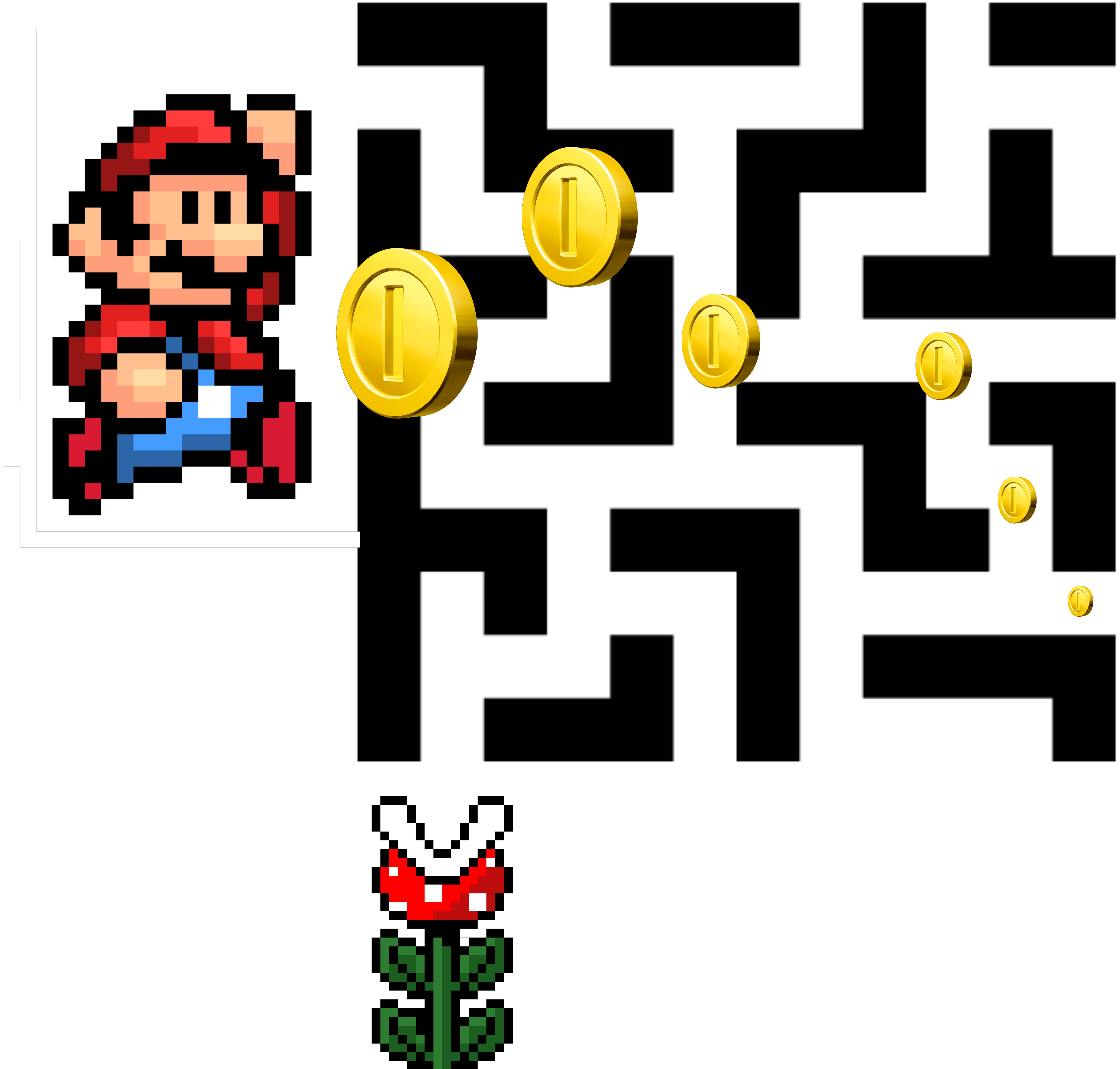




when future is uncertain

present > future



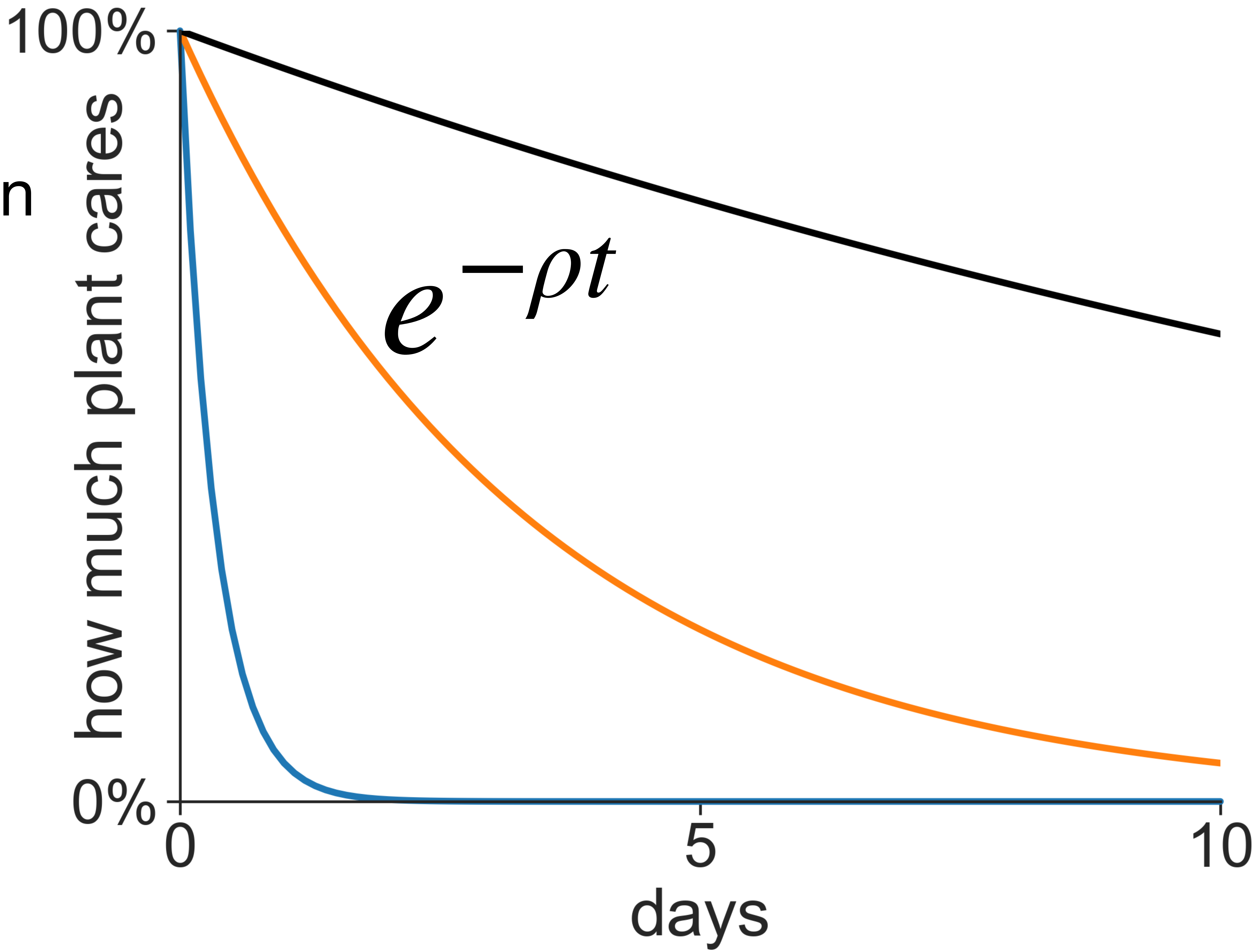
when future is uncertain

present > future

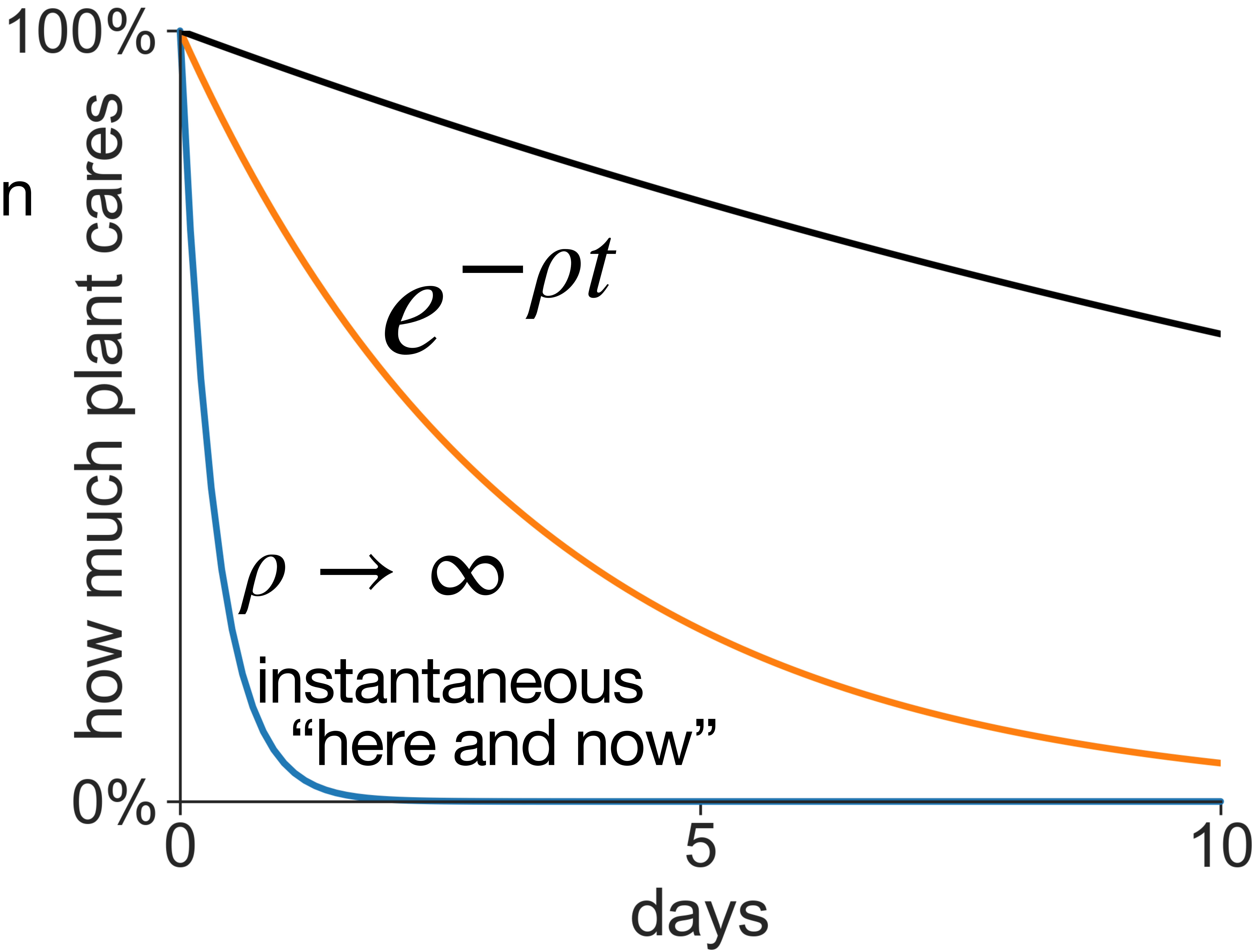
$$\int_0^\infty e^{-\rho t} A(g_s) dt$$

discount!

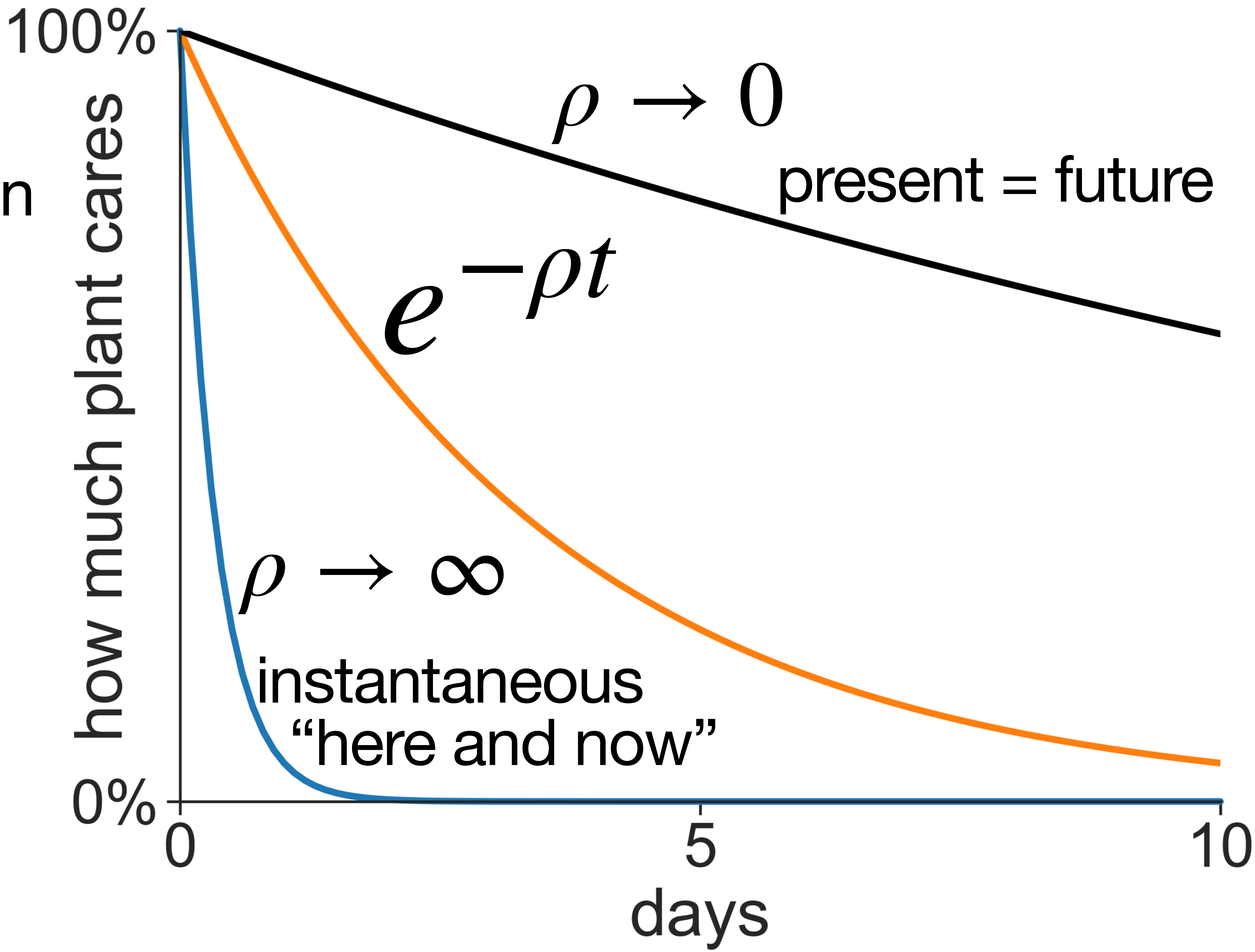
discount = horizon

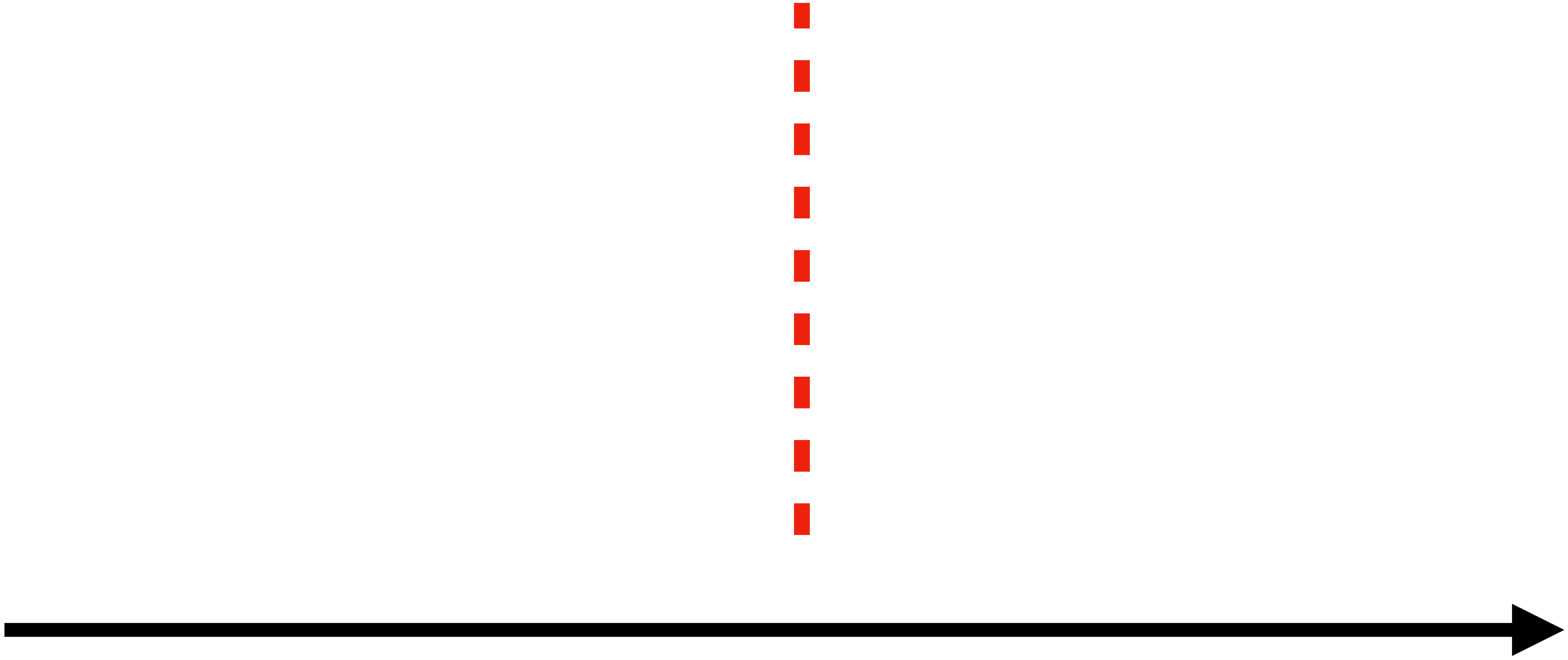


discount = horizon



discount = horizon





short time horizon

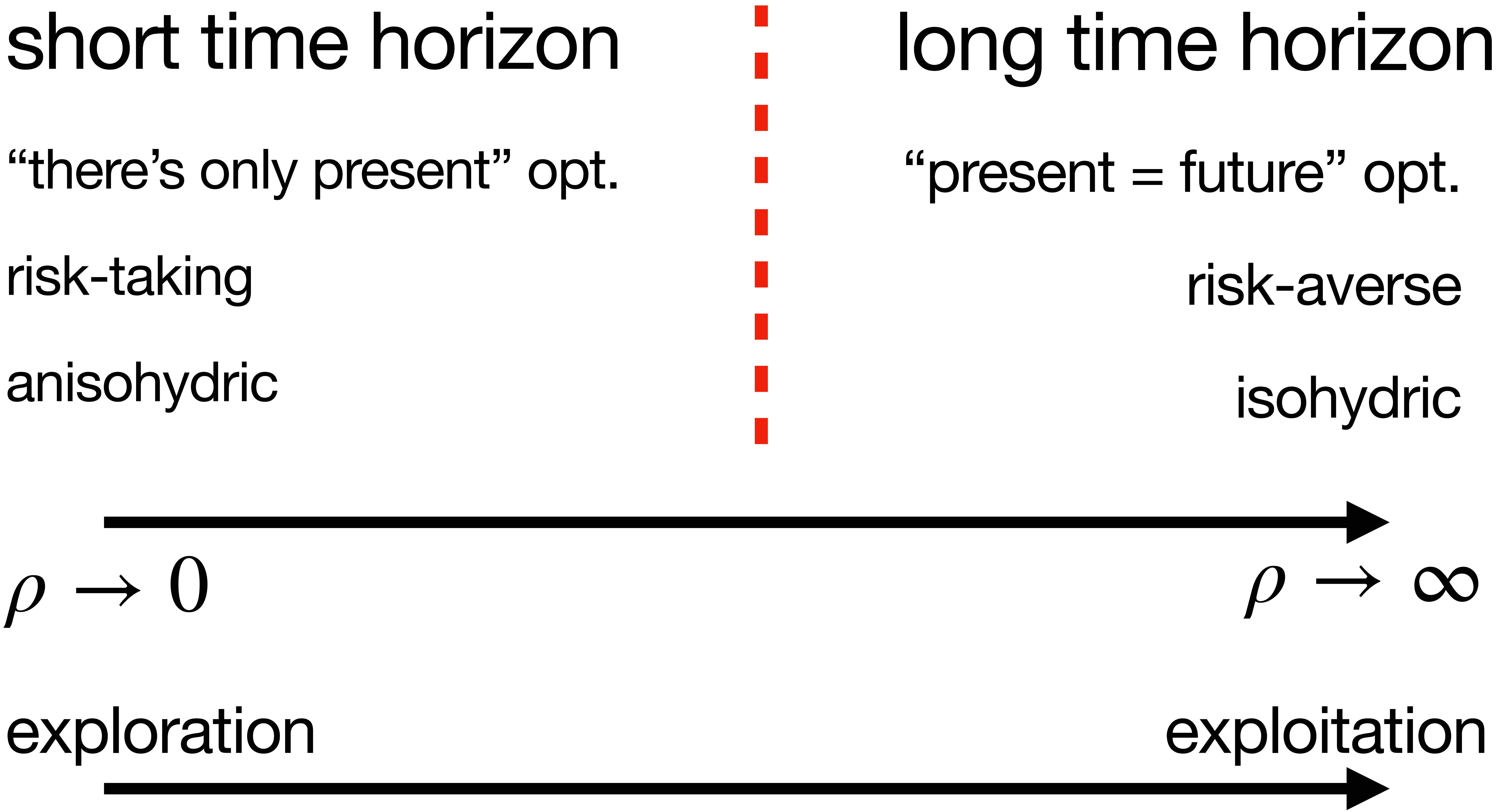
“there’s only present” opt.

risk-taking

anisohydric

$\rho \to 0$ $\rho \to \infty$

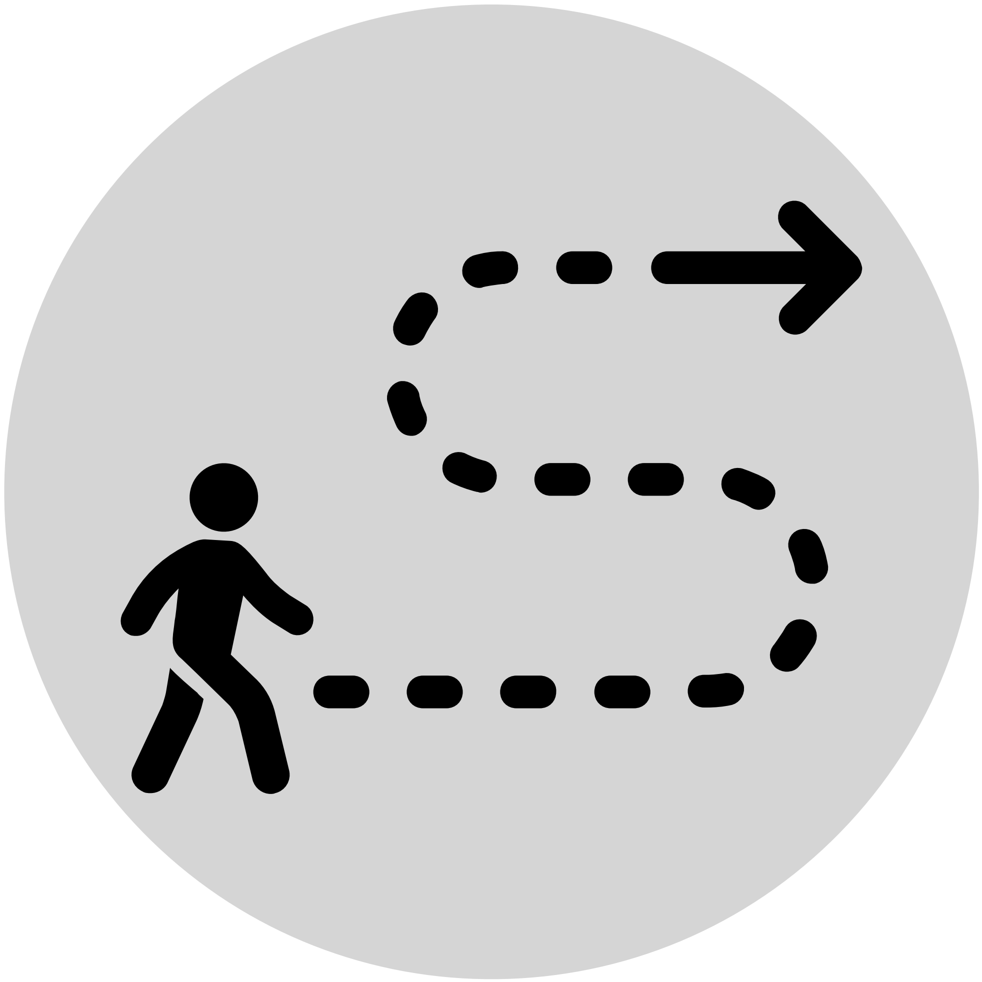
short time horizon	long time horizon
“there’s only present” opt.	“present = future” opt.
risk-taking	risk-averse
anisohydric	isohydric
$\rho \to 0$	$\rho \to \infty$



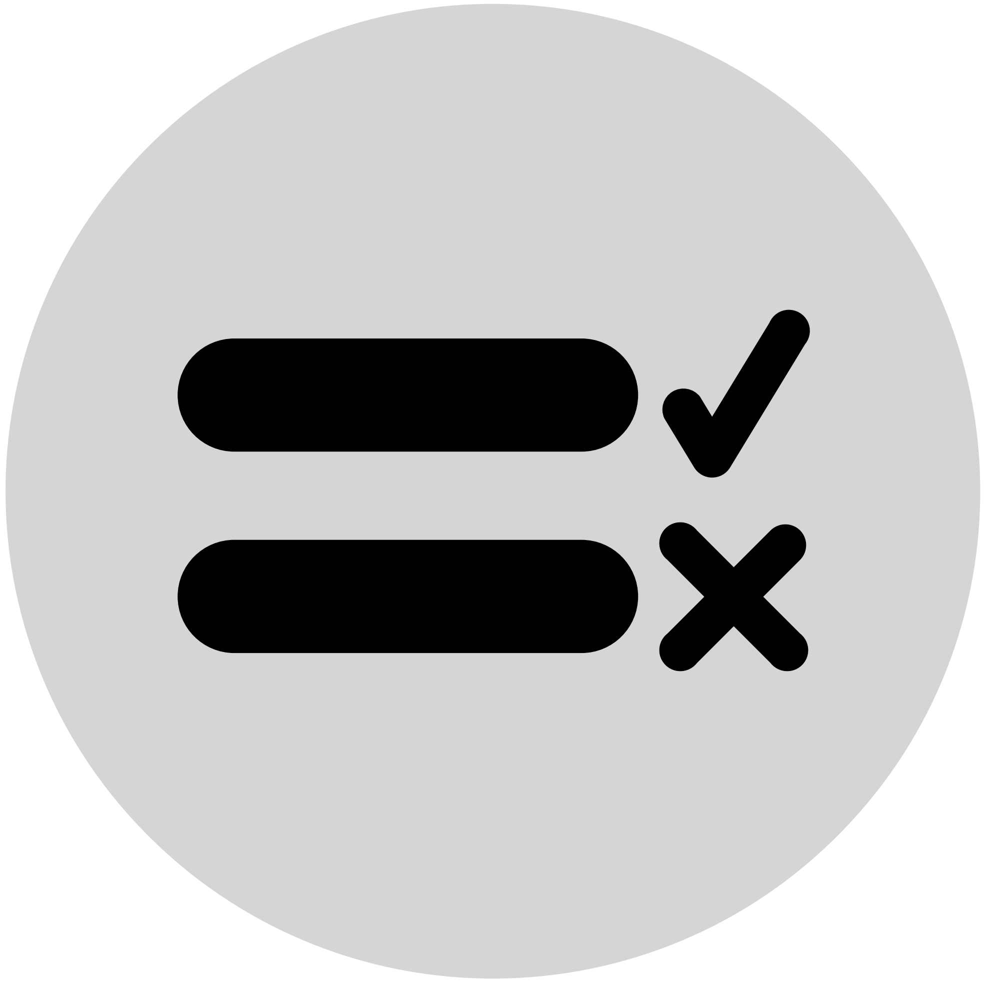
short time horizon
long time horizon
“there’s only present” opt.
“present = future” opt.
risk-taking
risk-averse
anisohydric
isohydric
$\rho \to 0$
$\rho \to \infty$
exploration
exploitation

take-home message

observed path



instantaneous rule



global principle