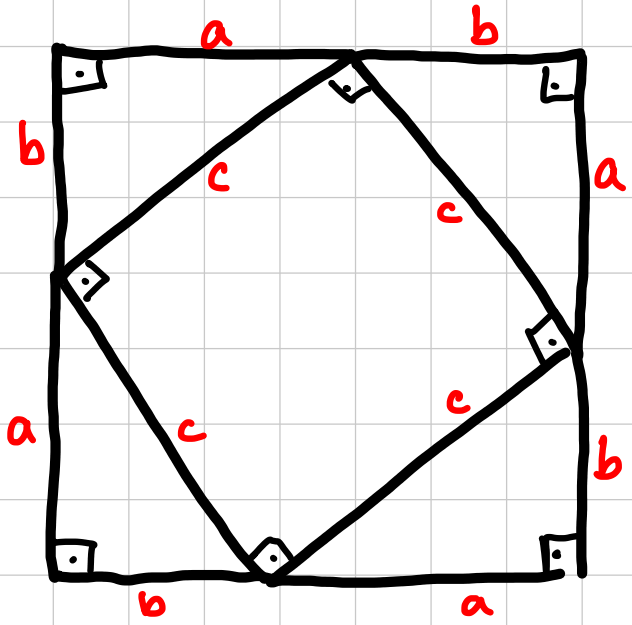
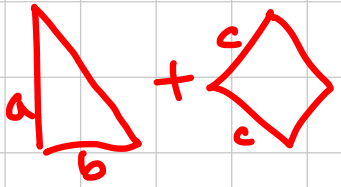
PROOFS OF THE PYTHAGOREAN THEOREM



AREA of BIG SQUARE $= (a+b)^2$

AREA of ALL ELEMENTS $= 4$  $+$

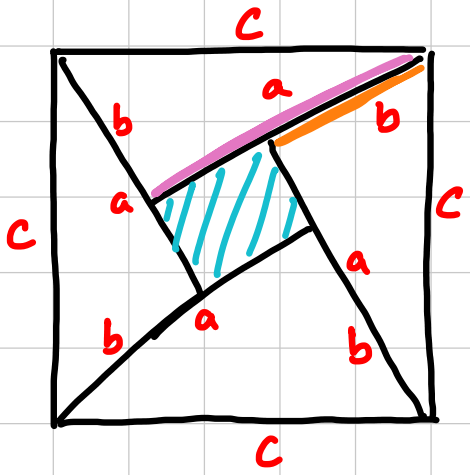
$= 4\frac{ab}{2} + c^2 = 2ab + c^2$

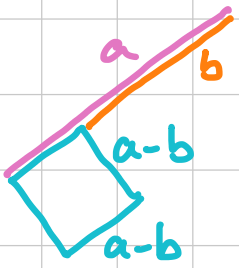
THEREFORE :

$$(a+b)^2 = 2ab + c^2$$

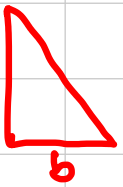
$$a^2 + 2ab + c^2 = 2ab + c^2$$

$$\boxed{a^2 + b^2 = c^2}$$





AREA OF BIG TRIANGLE $= c^2$

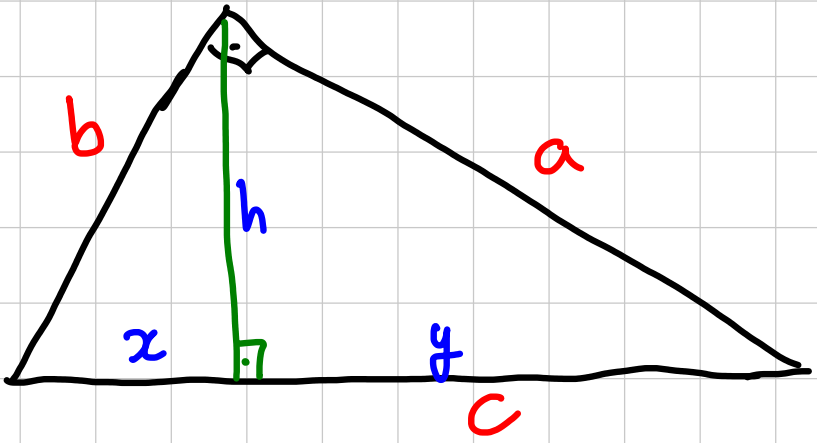
AREA OF ALL OTHER ELEMENTS $=$ 4  $+$ 

$$= 4\frac{ab}{2} + (a-b)^2$$

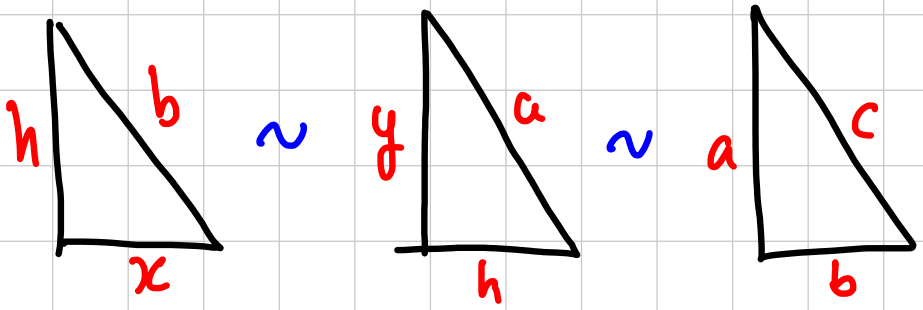
$$= \cancel{2ab} + a^2 - \cancel{2ab} + b^2$$

THEREFORE:

$$\boxed{a^2 + b^2 = c^2}$$



WE HAVE 3 SIMILAR TRIANGLES:



$$\frac{x}{b} = \frac{b}{c} \rightarrow x = \frac{b^2}{c}$$

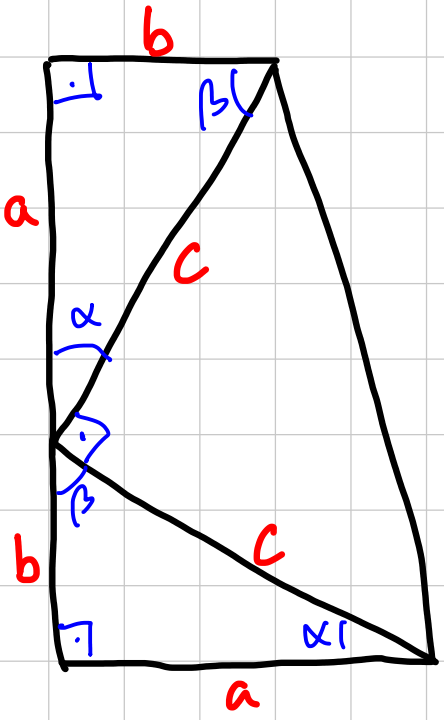
$$\frac{y}{a} = \frac{a}{c} \rightarrow y = \frac{a^2}{c}$$

WE KNOW THAT $\rightarrow$ $x + y = c$

THEREFORE

$$\frac{b^2}{c} + \frac{a^2}{c} = c$$

$$\Rightarrow \boxed{a^2 + b^2 = c^2}$$



WE MAKE A COPY OF THE TOP TRIANGLE  AND PUT IT ON THE BOTTOM

WE THEN CONNECT THE POINTS ON THE RIGHT

MEAN OF TOP AND BOTTOM

HEIGHT

AREA OF TRAPEZE: $\frac{(a+b)}{2} \cdot (a+b)$

AREA OF ALL ELEMENTS: $2\,\frac{a \cdot b}{2} + \frac{c^2}{2}$

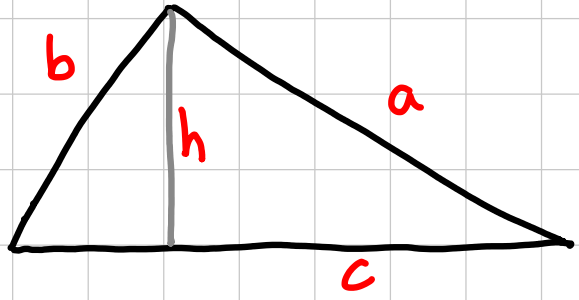
THEREFORE: $\frac{(a+b)^2}{2} = \frac{2ab}{2} + \frac{c^2}{2}$

$a^2 + 2ab + b^2 = 2ab + c^2$

$$\boxed{a^2 + b^2 = c^2}$$

PROOF BY JAMES GARFIELD

INVERSE PYTHAGORAS



AREA OF BIG TRIANGLE:

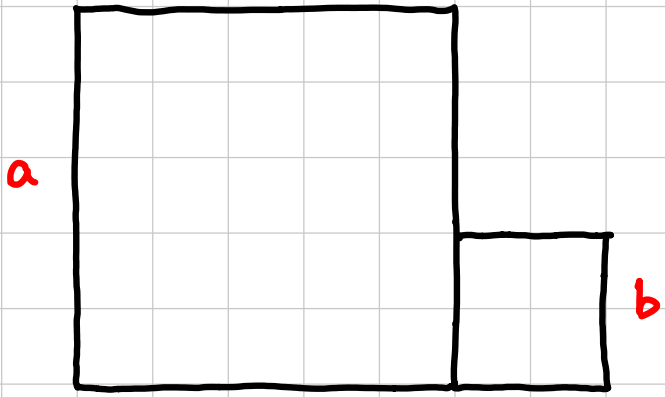
$$\frac{a \cdot b}{2} = \frac{c \cdot h}{2} \xrightarrow{\text{SQUARE}} a^2 b^2 = c^2 h^2$$

$$\rightarrow \frac{c^2}{a^2 b^2} = \frac{1}{h^2} \xrightarrow{\text{use } c^2 = a^2 + b^2} \frac{a^2 + b^2}{a^2 b^2} = \frac{1}{h^2}$$

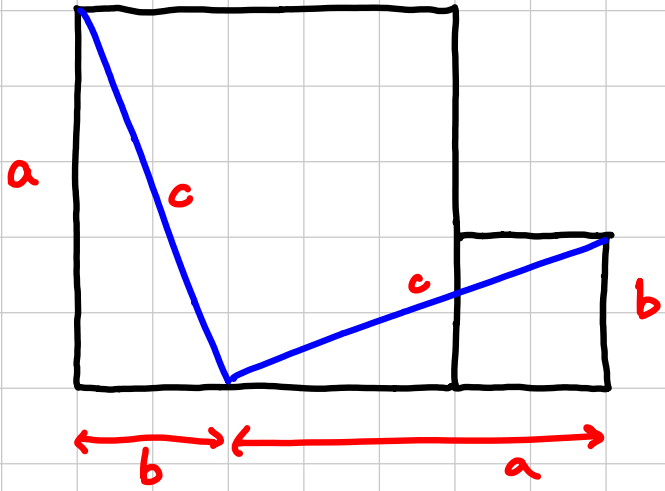
$$\rightarrow \frac{a^2}{a^2 b^2} + \frac{b^2}{a^2 b^2} = \frac{1}{h^2} \rightarrow \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{h^2}$$

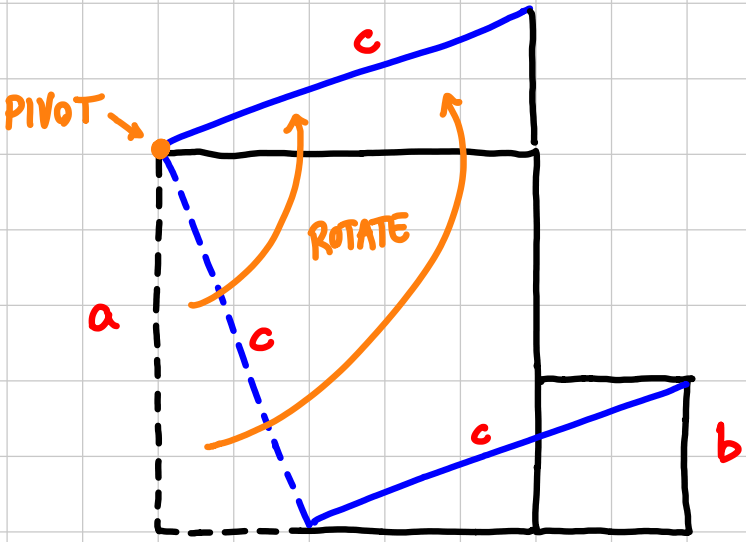
FINALLY:

$$\boxed{a^{-2} + b^{-2} = h^{-2}}$$

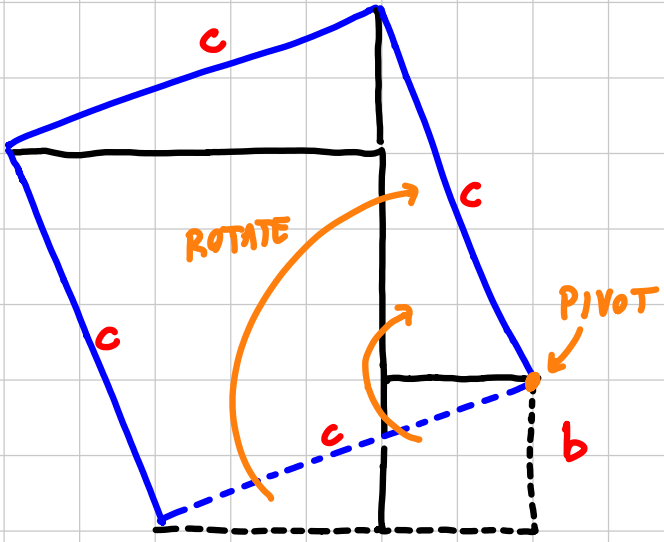


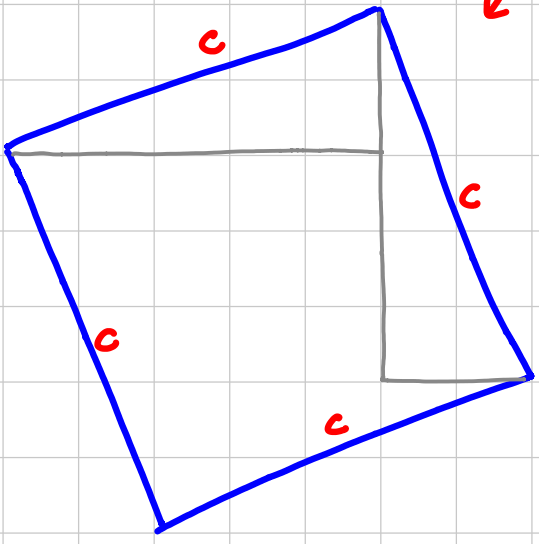
AREA = $a^2 + b^2$

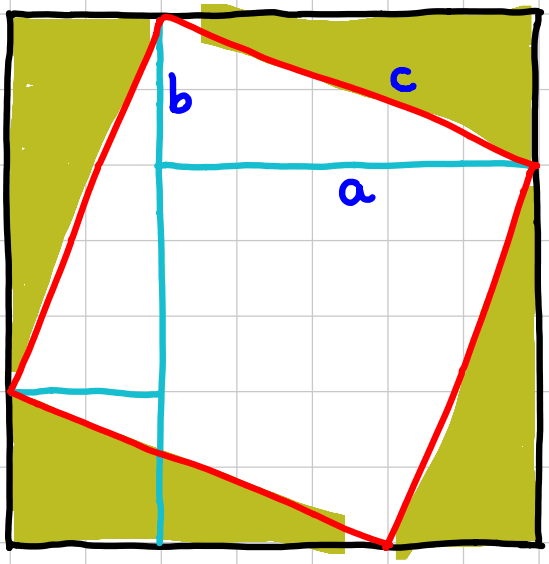




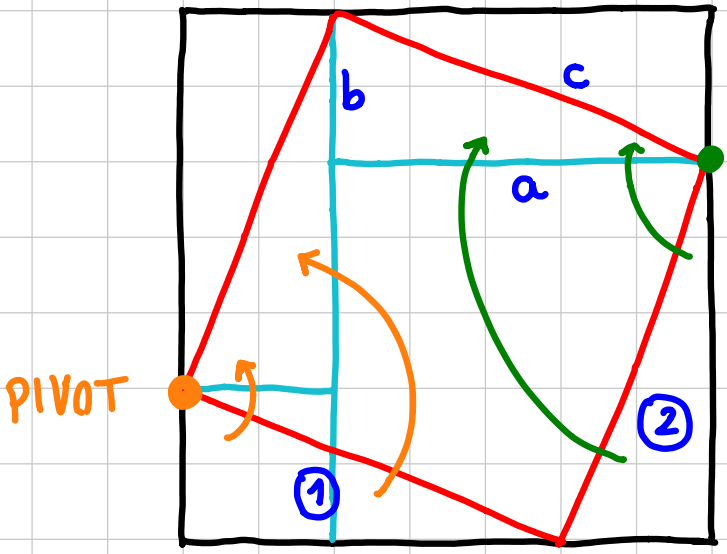
AREA = c^2



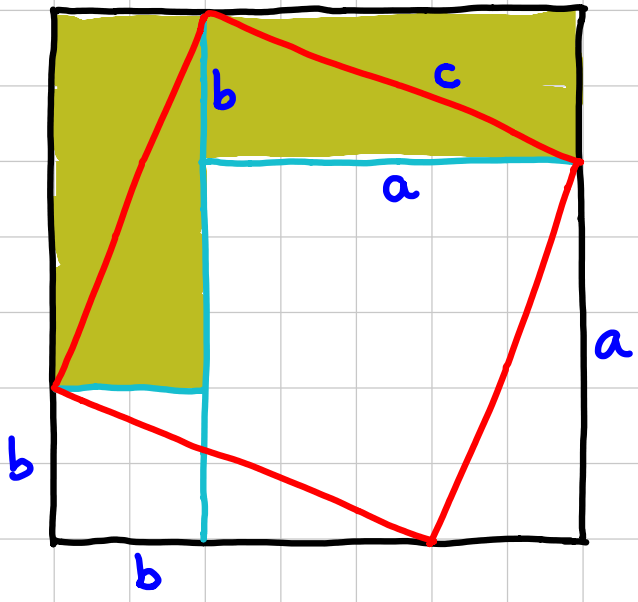




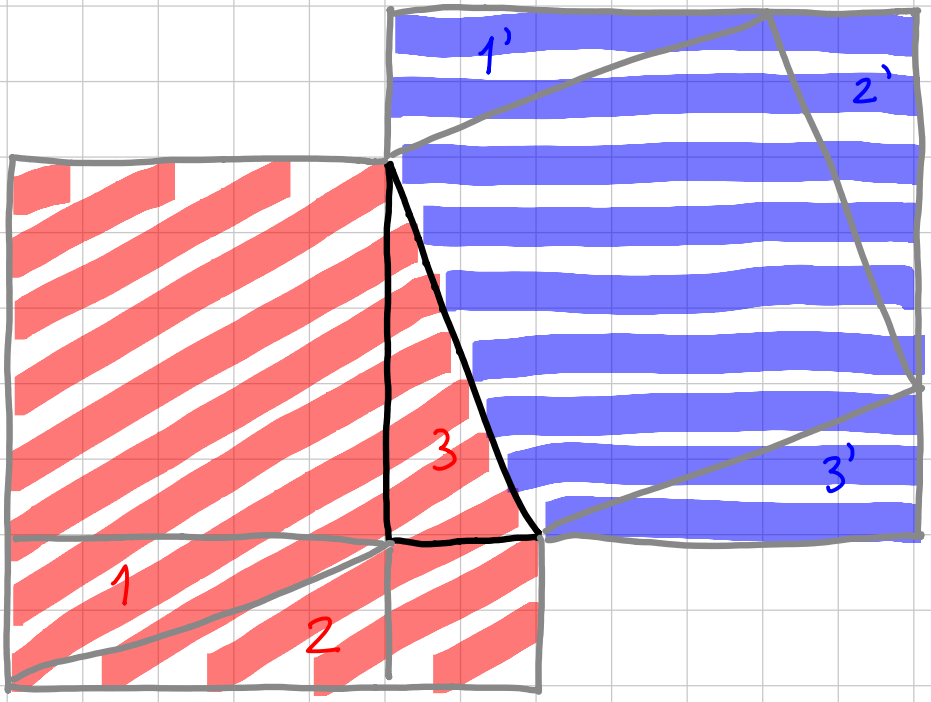
AREA IN WHITE = c^2



ROTATE BOTTOM LEFT (1) AND BOTTOM RIGHT (2) TRIANGLES

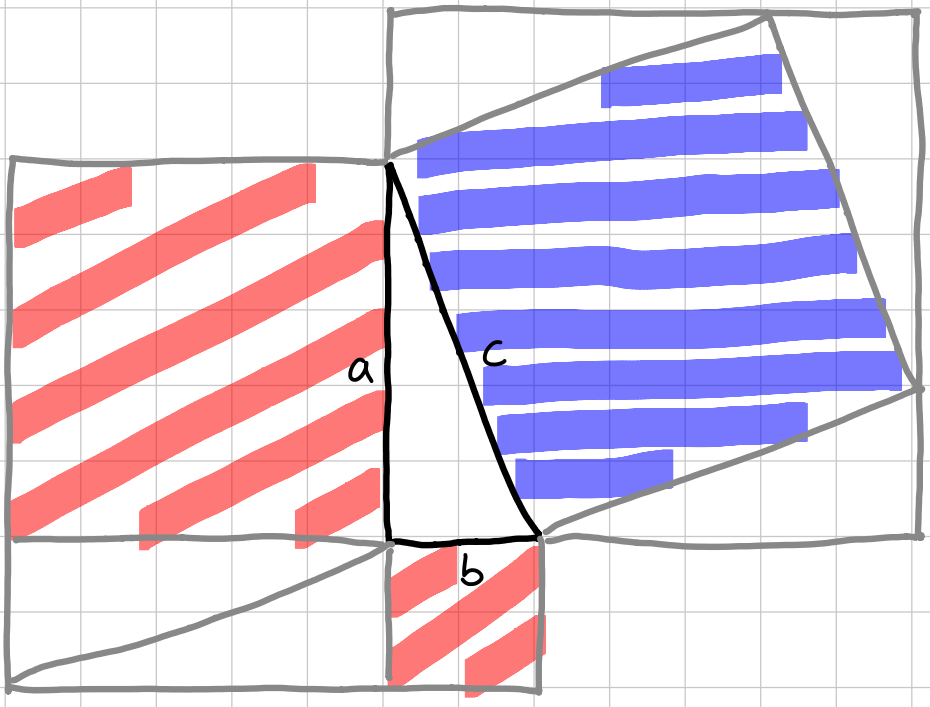


AREA IN WHITE = $a^2 + b^2$



THESE TWO PENTAGONAL SHAPES (RED AND BLUE) ARE IDENTICAL, THEREFORE THEY HAVE THE SAME AREA. REMOVE THE RED TRIANGLES (1, 2, 3) AND THE IDENTICAL BLUE TRIANGLES (1', 2', 3').

FINALLY:



$$a^2 + b^2 = c^2$$