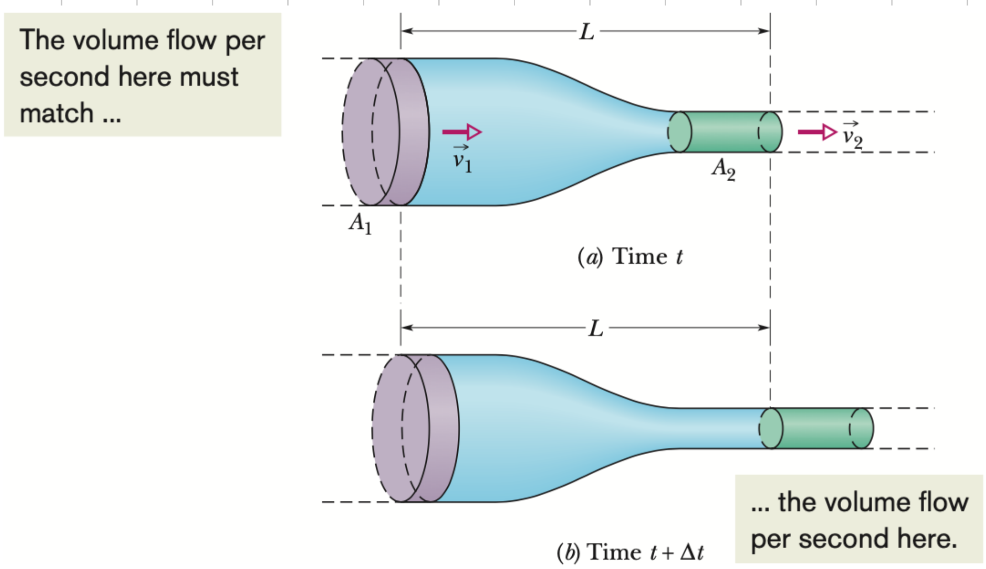
IDEAL FLUIDS

1. INCOMPRESSIBLE , $\rho = const$
2. NONVISCOUS , $\eta = 0$
3. LAMINAR

VOLUMETRIC FLOW RATE ספיקה



(a) Time t
(b) Time t + Δt

$$\frac{\Delta V_1}{\Delta t} = \frac{\Delta V_2}{\Delta t}$$

$$A_1 \frac{\Delta x_1}{\Delta t} = A_2 \frac{\Delta x_2}{\Delta t}$$

$$A_1 v_1 = A_2 v_2$$

$$Q = Av \quad (m^3/s)$$

ספיקה נפחית:

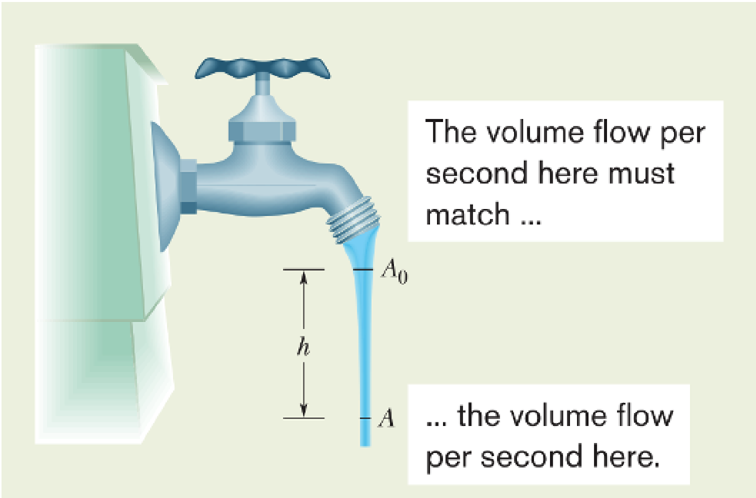
$$Q_1 = Q_2$$

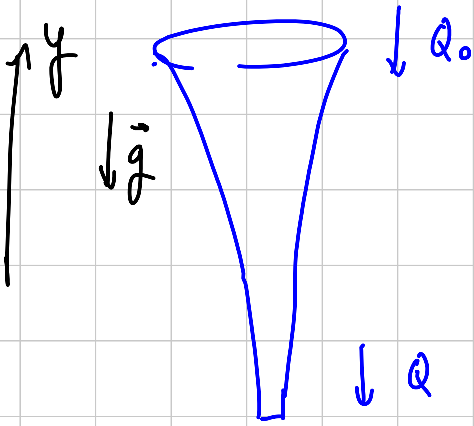
$$\rho = \frac{m}{V} \longrightarrow m = \rho V$$

Q_v ספיקה נפחית

Q_m ספיקה מסית

$$Q_m = \rho A v \quad (kg/s)$$





$$A_0 = 1.2\ \text{cm}^2$$
$$A = 0.35\ \text{cm}^2$$
$$h = 45\ mm$$
Volume flow rate $Av = ?$

$$Q = Q_0$$
$$A v = A_0 v_0 \qquad (1)$$

המים נופלים בנפילה חופשית, עם תאוצה g כלפי מטה:

$$v^2 = v_0^2 + 2a\Delta x \qquad \vec{a} = -g\hat{j}$$
$$v^2 = v_0^2 + 2(-g\hat{j})(-h\hat{j}) \qquad \Delta x = -h\hat{j}$$
$$v^2 = v_0^2 + 2gh \qquad (2)$$

מתוך (1):

$$v = v_0 \frac{A_0}{A}$$

נציב לתוך (2):

$$v_0^2 \left(\frac{A_0}{A}\right)^2 = v_0^2 + 2gh$$
$$v_0^2 \left[\left(\frac{A_0}{A}\right)^2 - 1\right] = 2gh$$
$$v_0^2 = \frac{2gh}{\left[\left(\frac{A_0}{A}\right)^2 - 1\right]} \rightarrow v_0 = \sqrt{\frac{2gh}{\left[\left(\frac{A_0}{A}\right)^2 - 1\right]}}$$

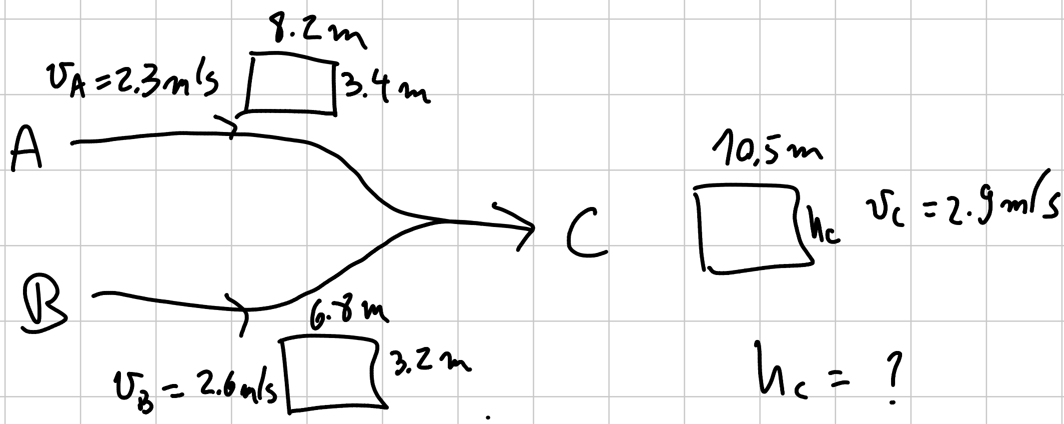
נציב את הנתונים:

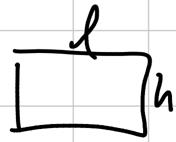
$$v_0 \simeq 0.286\ m/s$$
$$Q_0 = A_0 v_0 = 1.2\ cm^2 \cdot 0.286\ \frac{m}{s}$$

לכן

$$Q_0 = 1.2\ cm^2 \left(\frac{1m}{100\ cm}\right)^2 \cdot 0.286\ \frac{m}{s} = \frac{1.2 \cdot 0.286}{10^4}\ \frac{m^3}{s} = 3.4 \cdot 10^{-5}\ \frac{m^3}{s}$$
$$Q_0 = 1.2\ cm^2 \cdot 28.6\ \frac{cm}{s} = 34\ \frac{cm^3}{s} = 34\ \frac{mL}{s}$$

40. **שני נחלים**. שני נחלים מתמזגים ויוצרים נהר. רוחבו של נחל אחד הוא 8.2 מ', עומקו 3.4 מ' ומהירות הזרימה שלו 2.3 m/s. רוחבו של האחר הוא 6.8 מ', עומקו 3.2 מ' ומהירות זרימתו 2.3 m/s. רוחב הנהר הוא 10.5 מ', ומהירות הזרם בו היא 2.9 m/s. מהו עומקו?





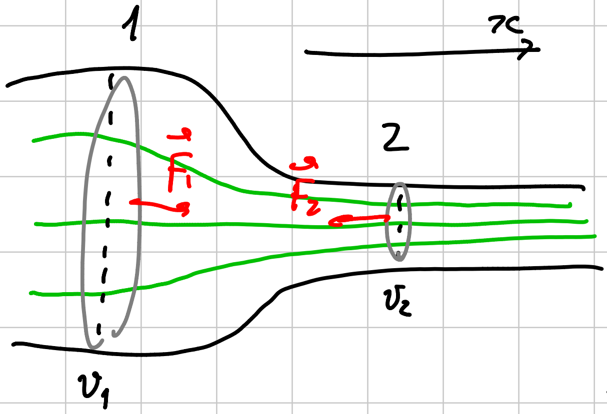
$v_A = 2.3 m/s$
$l_A = 8.2 m$
$h_A = 3.4 m$
$v_B = 2.6 m/s$
$l_B = 6.8 m$
$h_B = 3.2 m$
$v_c = 2.9 m/s$
$l_c = 10.5 m$

$$Q_A + Q_B = Q_C$$

$$v_A A_A + v_B A_B = v_c A_c$$

$$v_A h_A l_A + v_B h_B l_B = v_c h_c l_c$$

$$h_c = \frac{v_A h_A l_A + v_B h_B l_B}{v_c l_c} = 4.0 m$$



$v_1 < v_2$

הזורם מאיץ בין נקודה 1 לנקודה 2, לכן, מופעל עליו כוח.

הכוח הזה פועל על פני החתך A דרכו יש זרימה.

$$P = \frac{|F_1|}{A}$$

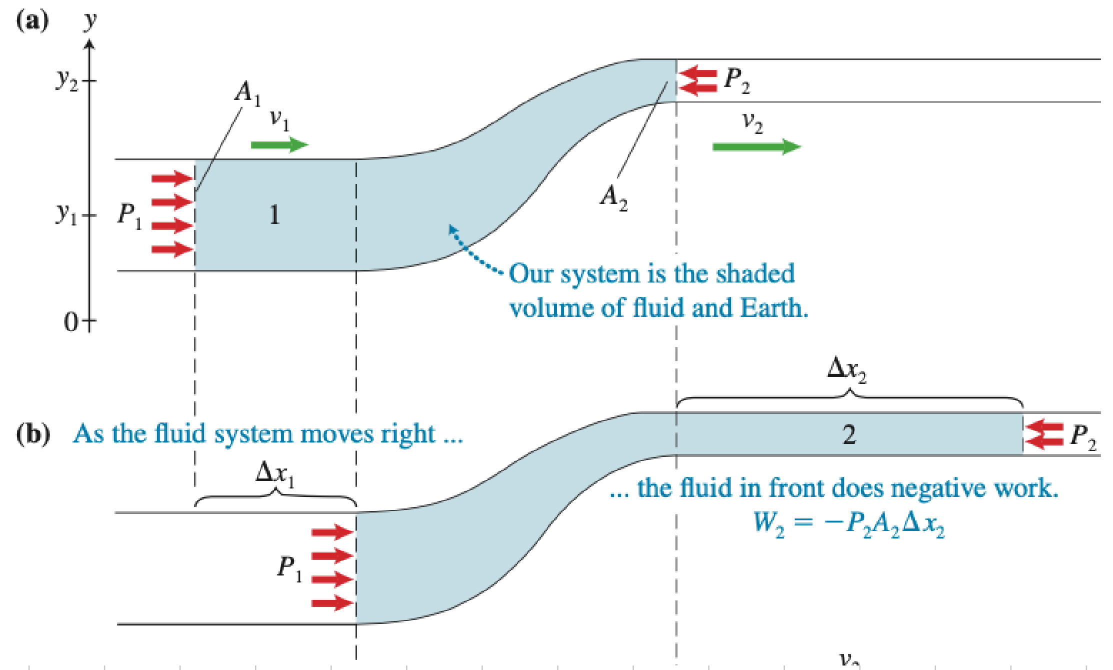
$$F_1 - F_2 > 0$$

$$P_1 A_1 - P_2 A_2 > 0$$

$$P_1 \frac{A_1}{A_2} > P_2 \quad , \quad \frac{A_1}{A_2} > 0$$

$$P_1 > P_2$$

BERNOULLI'S EQUATION



CONSERVATION OF ENERGY

$$E_1 + W^{NC} = E_2$$

$$E_2 - E_1 = W^{NC}$$

$$\Delta E = \Delta K + \Delta U = W^{NC}$$

$$\Delta K = \frac{m v_2^2}{2} - \frac{m v_1^2}{2} = \frac{m}{2}(v_2^2 - v_1^2)$$

$$\Delta U = mgy_2 - mgy_1 = mg(y_2 - y_1)$$

$$W_1 = F_1 \cdot \Delta x_1 \cdot \cos(0)$$

$$= P_1 A_1 \cdot \Delta x_1 = P_1 V$$

$$W_2 = F_2 \cdot \Delta x_2 \cdot \cos(\pi)$$

$$= - P_2 A_2 \cdot \Delta x_2 = - P_2 V$$

$$W = W_1 + W_2 = P_1 V - P_2 V = V(P_1 - P_2)$$

USING $\rho = \frac{m}{V} \longrightarrow V = \frac{m}{\rho}$

$$W = \frac{m}{\rho}(P_1 - P_2)$$

$$\frac{m}{\rho}(P_1-P_2)=\Delta E=\Delta K+\Delta U=\frac{m}{2}(v_2^2-v_1^2)+mg(y_2-y_1)$$

$$P_1-P_2=\frac{\rho v_2^2}{2}-\frac{\rho v_1^2}{2}+\rho g y_2-\rho g y_1$$

משוואת ברנולי

$$\boxed{P_1+\frac{\rho v_1^2}{2}+\rho g y_1=P_2+\frac{\rho v_2^2}{2}+\rho g y_2=\text{CONST}}$$

BERNOULLI'S EQ.

LOOKS LIKE: $K=\frac{mv^2}{2}$

LOOKS LIKE: $U=mgy$

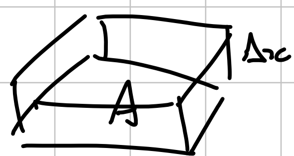
ENERGY DENSITIES

$$\frac{\rho v^2}{2}=\frac{m/V\, v^2}{2}=\frac{1}{V}\frac{mv^2}{2}=\frac{K}{V}$$ "DYNAMIC PRESSURE"

$$\rho g y=\frac{m}{V}gy=\frac{1}{V}mgy=\frac{U^{GRAV}}{V}$$ "WATER GRAV. POTENTIAL"

$$Pa=\frac{N}{m^2}\cdot\frac{m}{m}=\frac{J}{m^3}$$

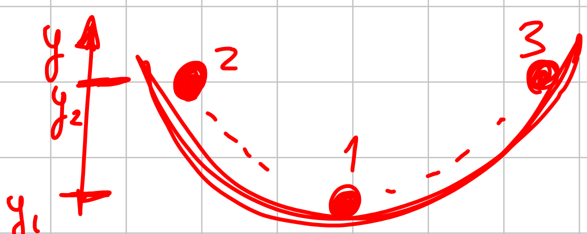
$$P(Pa)=\frac{F(N)}{A(m^2)}\frac{\Delta x(m)}{\Delta x(m)}=\frac{W(J)}{V(m^3)}$$



PSI $$\Psi=P+\frac{\rho v^2}{2}+\rho g y$$

פוטנציאל המים

Ψ גבוה $\rightarrow$ Ψ נמוך



צפיפות האנרגיה
הפוטנציאלית
ליחידת מסה

$$V = \frac{U}{m}$$

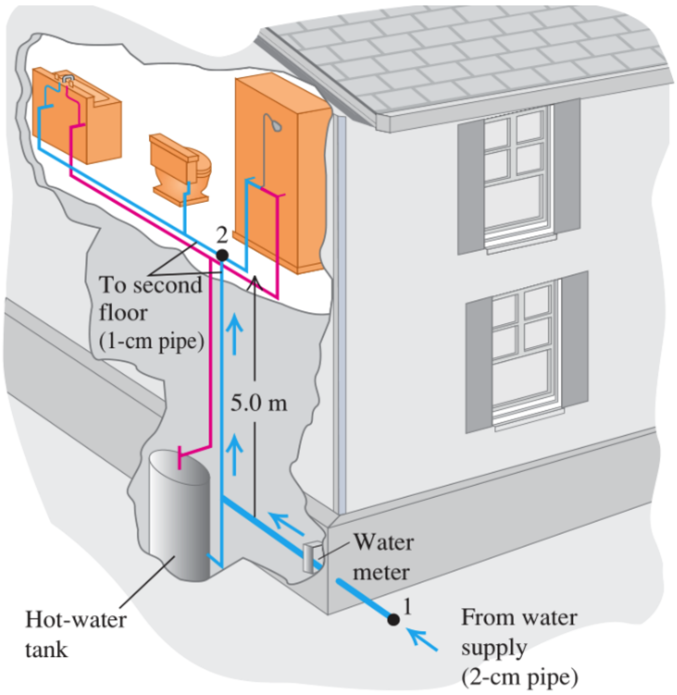
פוטנציאל

$$U \longrightarrow V$$

אנרגיה
פוטנציאלית

$$D_1 = 2\ cm$$
$$D_2 = 1\ cm$$
$$P_1 = 4.0 \times 10^5\ Pa$$
$$v_1 = 1.5\ m/s$$
$$h = 5\ m$$

$$v_2 = ?$$
$$P_2 = ?$$
$$A_2 v_2 = ?$$



$$Q_1 = Q_2$$
$$A_1 v_1 = A_2 v_2$$
$$\pi R_1^2 v_1 = \pi R_2^2 v_2$$
$$v_2 = v_1 \left(\frac{R_1}{R_2}\right)^2 = v_1 \left(\frac{D_1}{D_2}\right)^2 = 1.5 \left(\frac{2\,cm}{1\,cm}\right)^2 = \boxed{6\ m/s}$$

$$Q_2 = A_2 v_2 = A_1 v_1 = \pi \left(\frac{D_1}{2}\right)^2 v_1 = \pi \left(\frac{2 \cdot 10^{-2}}{2}\right)^2 1.5 = \pi \cdot 10^{-4} \cdot 1.5$$

$$A_2 v_2 = 0.47 \cdot 10^{-3} \frac{m^3}{s} \left(\frac{1000\,L}{1\,m^3}\right) \rightarrow A_2 v_2 = \boxed{0.47\ \frac{L}{s}}$$

$$P_2 + \frac{\rho v_2^2}{2} + \rho g y_2 = P_1 + \frac{\rho v_1^2}{2} + \rho g y_1$$

$$P_2 = P_1 + \frac{\rho}{2}(v_1^2 - v_2^2) + \rho g (y_1 - y_2)$$

$$P_2 = 4 \cdot 10^5 + \frac{10^3}{2}(1.5^2 - 6^2) + 10^3 \cdot 9.8(-5)$$

$$P_2 = 3.34 \cdot 10^5\ Pa \qquad \frac{P_2}{P_1} = 83\%$$

$$v_2 = ?$$

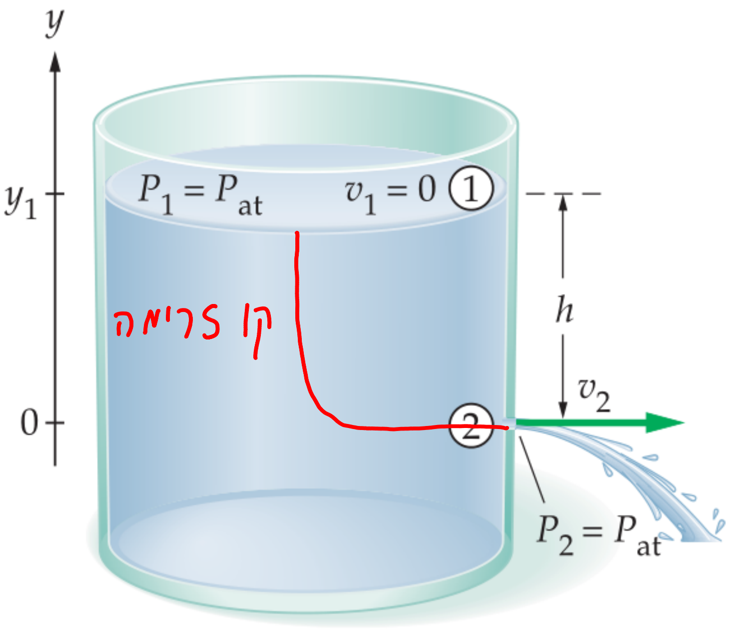
$$P_1 + \rho g y_1 + \frac{\rho v_1^2}{2} = P_2 + \rho g y_2 + \frac{\rho v_2^2}{2}$$

$P_1 = P_2 = P_{ATM}$

$y_2 = 0$

$v_1 = 0$

$$\rho g y_1 = \frac{\rho v_2^2}{2} \rightarrow \boxed{v_2 = \sqrt{2 g y_1}}$$



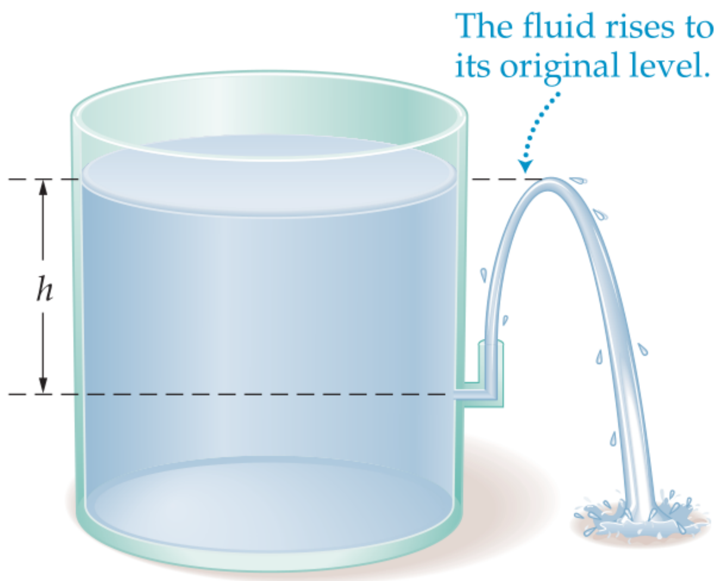
אם הפיה מופנית כלפי מעלה, אז המים יוכלו להגיע לגובה מפלס המים של המיכל!
משוואת ברנולי נובעת משימור אנרגיה, אז התוצאה לא מפתיעה כל-כך...

$U_2 + K_2 = U_1 + K_1$

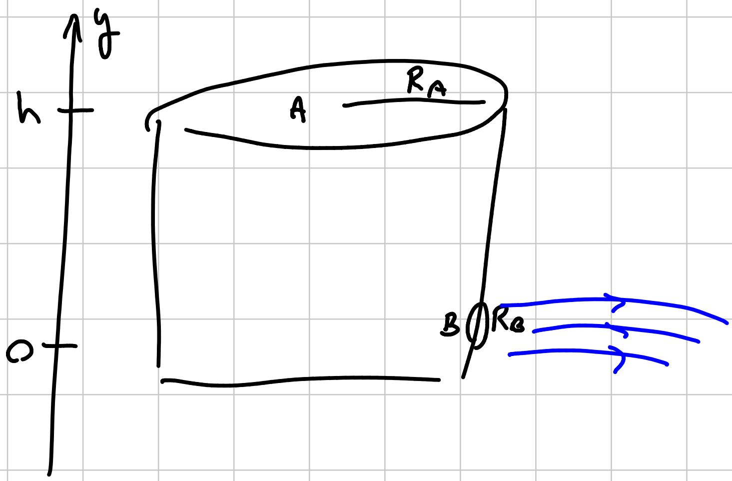
$$\frac{m v_2^2}{2} = m g y$$

$$y = \frac{v_2^2}{2g} = \frac{1}{2g}\left(\sqrt{2 g y_1}\right)^2$$

$$y = y_1$$



פתרון אחר



$$P_A + \rho g y_A + \frac{\rho v_A^2}{2} = P_B + \rho g y_B + \frac{\rho v_B^2}{2}$$

$$P_A = P_{ATM}$$

$$y_B = 0 \; ; \; y_A = h$$

$$Q_A = Q_B$$

$$v_A A_A = v_B A_B$$

$$v_A \pi R_A^2 = v_B \pi R_B^2$$

$$v_A = v_B \frac{R_B^2}{R_A^2}$$

$$P_{ATM} + \rho g h + \frac{\rho}{2}\left(\frac{v_B R_B^2}{R_A^2}\right)^2 = P_B + 0 + \frac{\rho}{2} v_B^2$$

$$(P_{ATM} - P_B) + \rho g h = v_B^2 \frac{\rho}{2}\left[1 - \left(\frac{R_B}{R_A}\right)^4\right]$$

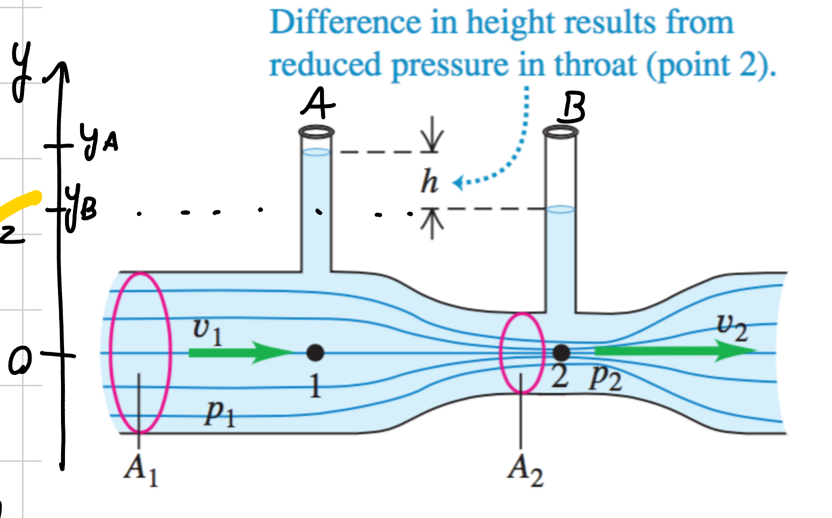
$$v_B^2 = 2\left[\frac{(P_{ATM} - P_B)}{\rho} + gh\right]\left[1 - \left(\frac{R_B}{R_A}\right)^4\right]^{-1}$$

$v_1 = ?$

KNOWN: A_1, A_2, h, g

$$P_1 + \frac{\rho v_1^2}{2} + \rho g y_1 = P_2 + \frac{\rho v_2^2}{2} + \rho g y_2$$

$$y_1 = y_2$$



$$\left.\begin{aligned} P_1 &= P_{ATM} + \rho g y_A \\ P_2 &= P_{ATM} + \rho g y_B \end{aligned}\right\} \quad y_A - y_B = h$$

$$P_{ATM} + \rho g y_A + \frac{\rho v_1^2}{2} = P_{ATM} + \rho g y_B + \frac{\rho v_2^2}{2}$$

$$\rho g (y_A - y_B) = \frac{\rho}{2}(v_2^2 - v_1^2)$$

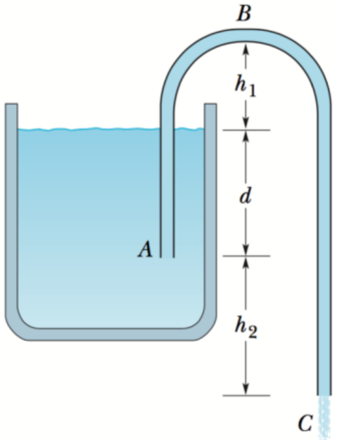
$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2} v_1$$

$$2gh = \left[\left(\frac{A_1}{A_2}\right)^2 - 1\right] v_1^2$$

$$v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

83 Figure 14-56 shows a *siphon,* which is a device for removing liquid from a container. Tube *ABC* must initially be filled, but once this has been done, liquid will flow through the tube until the liquid surface in the container is level with the tube opening at *A*. The liquid has density 1000 kg/m³ and negligible viscosity. The distances shown are $h_1 = 25$ cm, $d = 12$ cm, and $h_2 = 40$ cm. (a) With what speed does the liquid emerge from the tube at *C*? (b) If the atmospheric pressure is 1.0×10^5 Pa, what is the pressure in the liquid at the topmost point *B*? (c) Theoretically, what is the greatest possible height h_1 that a siphon can lift water?



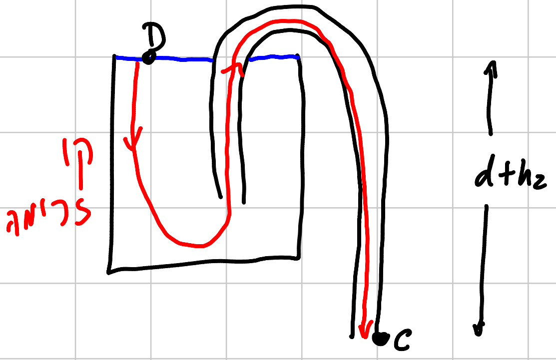
(a)

$\rho = 1000\ kg/m^3$

$h_1 = 25\ cm = 0.25\ m$

$h_2 = 40\ cm = 0.40\ m$

$d = 12\ cm = 0.12\ m$



$v_c = ?$

$$P_D + \frac{\rho v_D^2}{2} + \rho g y_D = P_c + \frac{\rho v_c^2}{2} + \rho g y_c$$

$P_c = P_D = P_{ATM}$

$v_D = 0$

$y_D - y_c = d + h_2$

$$\rho g(d+h_2) = \frac{\rho v_c^2}{2}$$

$$v_c = \sqrt{2g(d+h_2)}$$

(b)

$P_{ATM} = P_D = 1.0 \cdot 10^5 Pa$

$v_B = v_c$

$$P_D + \rho g y_D + \frac{\rho v_D^2}{2} = P_B + \rho g y_B + \frac{\rho v_B^2}{2}$$

$y_B - y_D = h_1$

$v_D = 0$

$$P_B = P_D - \rho g(y_B - y_D) - \frac{\rho v_B^2}{2} = P_D - \rho g h_1 - \frac{\rho}{2} \cdot 2g(d+h_2)$$

$$P_B = P_D - g\rho\left[h_1 + d + h_2\right]$$

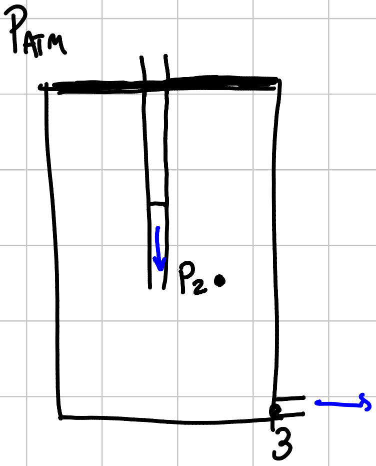
$$P_B = 1.0 \cdot 10^5 - 10^3 \cdot 9.8(0.77) = 1.0 \cdot 10^5\left(1 - \frac{9.8}{100} \cdot 0.77\right)$$

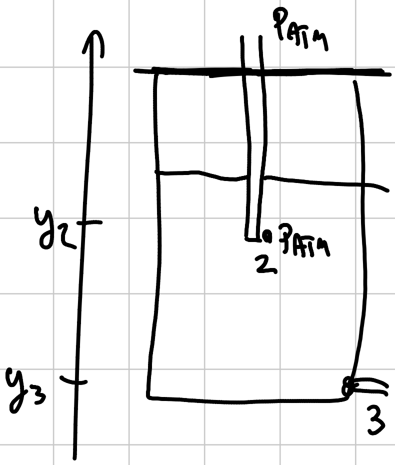
$$P_B = 1.0 \cdot 10^5 \cdot 0.925$$

(c)

$$P_B = 0 = 1.0 \cdot 10^5\left(1 - \frac{9}{100} \cdot h_1\right) \rightarrow \frac{9}{100} \cdot h_1 = 1 \rightarrow h_1 = \frac{100}{9} = 10.2\ m$$

MARIOTTE'S BOTTLE





$$P_2 + \rho g y_2 + \frac{\rho v_2^2}{2} = P_3 + \rho g y_3 + \frac{\rho v_3^2}{2}$$

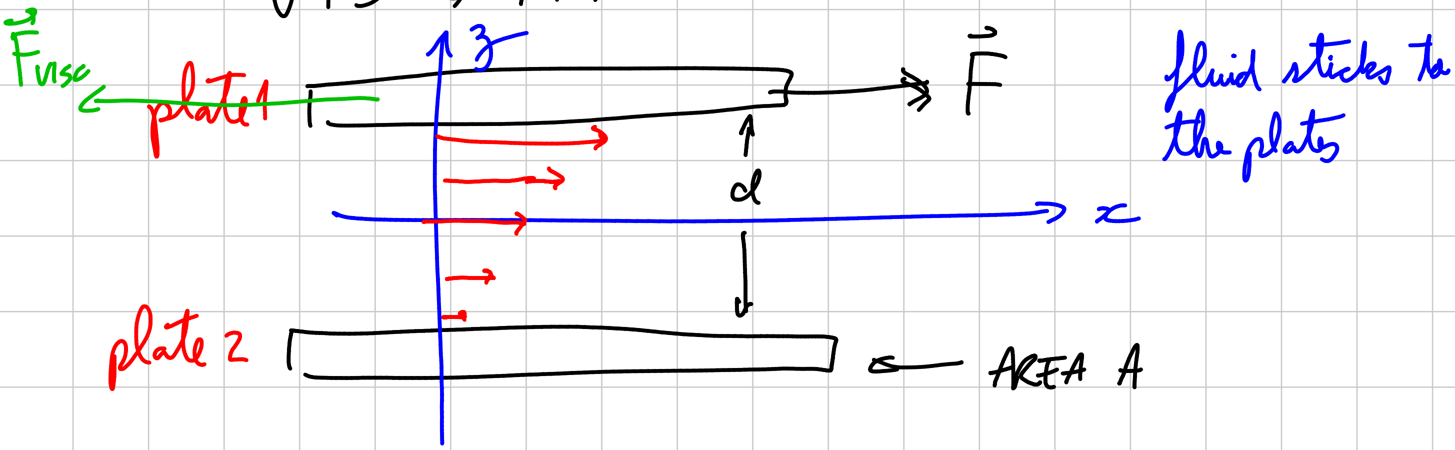
$P_2 = P_3 = P_{ATM}$, $v_2 << 1 \sim 0$

$$v_3^2 = 2g(y_2 - y_3)$$

$$v_3 = \sqrt{2g(y_2 - y_3)} = \text{CONST}$$

$$Q_3 = v_3 \cdot A_3 = \text{CONST}$$

VISCOSITY



$$\left.\begin{array}{l} v \propto F \\ v \propto d \\ v \propto \frac{1}{A} \end{array}\right\} \quad v \propto \frac{Fd}{A}$$

$$F \propto \frac{vA}{d}$$

$$F = \eta \frac{vA}{d}$$

$$[\eta] = \frac{[F][d]}{[v][A]} = \frac{M L T^{-2} L}{L T^{-1} L^2} = M L^{-1} T^{-1}$$

$$\eta = \frac{F}{A}\frac{d}{v} \quad (Pa \cdot s) \quad ; \quad Pa \cdot s = kg\, m^{-1} s^{-1} \qquad SI$$

τ
מאמץ גזירה
SHEAR STRESS (→ $\frac{F}{A}$)

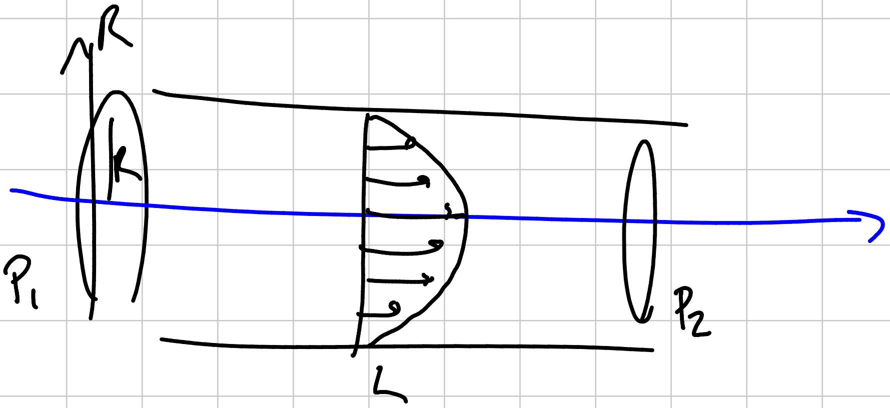
$$poise = g \cdot cm^{-1} s^{-1} \qquad cgs$$

$$cP = m\,Pa\,s$$

centi poise = milli Pascal second

η STRONGLY DEPENDS ON THE MATERIAL AND ON TEMPERATURE. AS T GOES UP, LIQUIDS GET RUNNIER, WHILE GASES GET THICKER

CYLINDRICAL PIPE - POISEUILLE'S LAW



$$v_{MAX} = \frac{(P_1 - P_2) R^2}{4 \eta L}$$

$$v(r) = v_{MAX}\left(1 - \frac{r^2}{R^2}\right) = \frac{(P_1 - P_2) R^2}{4 \eta L}\left(1 - \frac{r^2}{R^2}\right)$$

$$Q = \frac{(P_1 - P_2) \pi R^4}{8 \eta L}$$

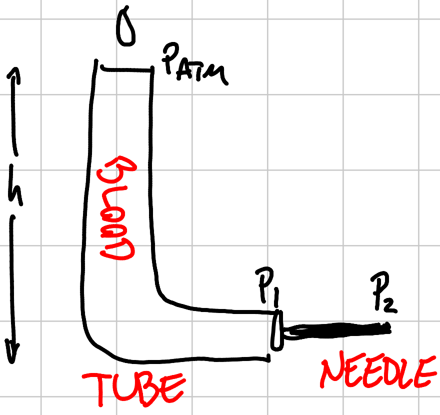
POISEUILLE'S LAW

חוק פואסיי

EXAMPLE

Example 9.2 In giving a transfusion, blood drips from a sealed storage bag with a 1 m pressure head through capillary tubing of 2 mm inside diameter, passing through a hypodermic needle that is 4 cm long and has an inside diameter of 0.5 mm. If the blood pressure within the vein into which the blood is being transfused is at a gauge pressure of 18 torr, find how long it will take to give the patient 1 L of blood. How long will it take if the inside diameter of the needle is only 0.4 mm?

NEWMAN, PHYSICS OF THE LIFE SCIENCES



$$P_1 = P_{ATM} + \rho g h$$

$$P_2 = P_{ATM} + 18\,torr$$

$$P_1 - P_2 = \rho g h - 18\,torr\left(\frac{1\cdot 10^5 Pa}{760\,torr}\right) = \rho g h - \frac{18\cdot 10^5}{760}$$

$$\rho_{BLOOD} = 1.06\cdot 10^3\ kg/m^3$$

$$\eta_{BLOOD} = 4\cdot 10^{-3}\ Pa\cdot s$$

$$h = 1.0\,m$$

$$R = \frac{0.5\cdot 10^{-3}}{2} m, \quad L = 4\cdot 10^{-2} cm$$

$$Q = \frac{(P_1 - P_2)\pi R^4}{8\eta L}$$

$$Q = \frac{\left[1.06\cdot 10^{-3}\cdot 9.8\cdot 1.0 - \frac{18}{760}\cdot 10^5\right]\pi\left(0.25\cdot 10^{-3}\right)^4}{8\cdot 4\cdot 10^{-3}\cdot 4\cdot 10^{-2}} = 7.7\cdot 10^{-8} m^3/s$$

$$7.7\cdot 10^{-8} m^3 = 1s$$

$$1L = 10^{-3} m^3 = 10^{-3} m^3\left(\frac{1s}{7.7\cdot 10^{-8} m^3}\right) = \frac{10^{-3}}{7.7\cdot 10^{-8}} s \approx 13\cdot 10^3 s = 3.6\,h$$

IF $R = \frac{0.4\cdot 10^{-3}}{2} m$:

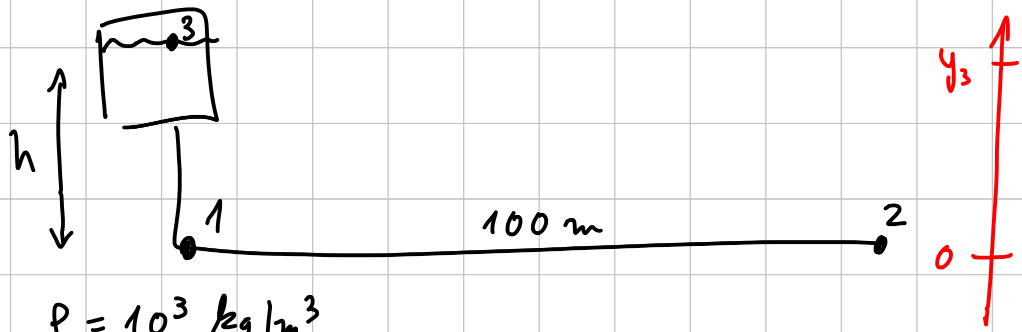
$$Q = 3.1\cdot 10^{-8} m^3/s$$

$$1L = 10^{-3} m^3\left(\frac{1s}{3.1\cdot 10^{-8} m^3}\right) = 8.8\,h$$

מים זורמים ממכל גדול, שגובהו מטר אחד, לתוך צינור הנמצא בתחתית המכל.
קוטרו הפנימי של הצינור הוא 1 ס"מ, ואורכו 100 מטר. צמיגות המים שווה ל-
1.0×10^{-3} Pa s.

א. יש להשוות את הספיקה המתקבלת לפי משוואת ברנולי לספיקה המתקבלת לפי נוסחת פואסיי (בהנחה שהפרש הלחצים נקבע לפי גובה המים).

ב. מהי צפיפות האנרגיה הקינטית של מסת המים היוצאת מהצינור, ומה שיעור ההפסד באנרגיה של מסה זאת, בזרימה קבועה?



א

$\rho = 10^3\ kg/m^3$

$d = 1cm \rightarrow R = 0.5\cdot 10^{-2} m$

$L = 100\ m$

$\eta = 1.0\cdot 10^{-3}\ Pa\cdot s$

$h = 1.0\ m$

ברנולי:

$$P_3 + \rho g y_3 + \frac{\rho v_3^2}{2} = P_2 + \rho g y_2 + \frac{\rho v_2^2}{2}$$

$P_2 = P_3 = P_{ATM}$

$y_3 = h\ ;\ y_2 = 0$

$v_3 = 0$

$$\rho g y_3 = \frac{\rho v_2^2}{2} \longrightarrow v_2^2 = 2gh$$

$$v_2 = \sqrt{2gh}$$

$$Q_2 = A_2 v_2 = \pi R^2 \cdot v_2 = \pi R^2 \sqrt{2gh}$$

$$Q_2 = 0.348\cdot 10^{-3} \frac{m^3}{s} = 0.348 \frac{L}{s} = 348 \frac{mL}{s}$$

פואסיי

$P_1 = P_{ATM} + \rho g h$

$P_2 = P_{ATM}$

$P_1 - P_2 = \rho g h$

$$Q = \frac{(P_1 - P_2)\pi R^4}{8\eta L}$$

$$Q = \frac{\rho g h \cdot \pi \cdot R^4}{8\eta L} \longrightarrow Q = 0.024\cdot 10^{-3}\frac{m^3}{s} = 0.024\frac{L}{s} = 24\frac{mL}{s}$$

1

ברנולי

$$Q_B = A v_B \rightarrow v_B = \frac{1}{A} Q_B = \frac{348 \frac{mL}{s}}{A} \qquad \frac{\rho v_B^2}{2} = \frac{\rho}{2}\left(\frac{348 \frac{mL}{s}}{A}\right)^2$$

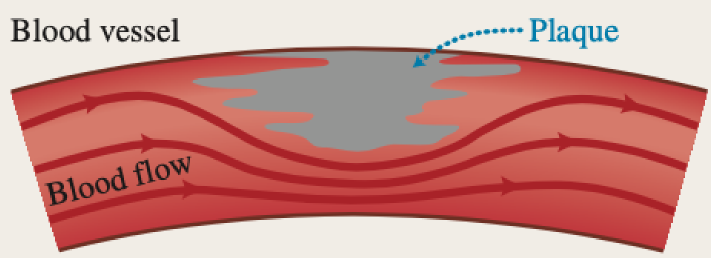
פלס״

$$Q_p = A v_p \rightarrow v_p = \frac{1}{A} Q_p = \frac{24 \frac{mL}{s}}{A} \qquad \frac{\rho v_p^2}{2} = \frac{\rho}{2}\left(\frac{24 \frac{mL}{s}}{A}\right)^2$$

$$\frac{\rho v_p^2}{2} \Big/ \frac{\rho v_B^2}{2} = \frac{\left(24 \frac{mL}{s}/A\right)^2}{\left(348 \frac{mL}{s}/A\right)^2} = \left(\frac{24}{348}\right)^2 = 0.0094 = 0.94\%$$

הפסד
של 99%
של האנרג׳ה

Because of plaque buildup, the radius of an artery in a person's heart decreases by 40%. Determine the ratio of the present flow rate to the original flow rate if the pressure across the artery, its length, and the viscosity of blood are unchanged.



תרגול

$R_0 =$ רדיוס מקורי

$$R = \frac{60}{100} R_0 = \beta R_0$$

$$\frac{Q}{Q_0} = \frac{\frac{(P_1 - P_2)\pi R^4}{8\eta L}}{\frac{(P_1 - P_2)\pi R_0^4}{8\eta L}} = \frac{R^4}{R_0^4} = \left(\frac{R}{R_0}\right)^4 = \left(\frac{\beta R_0}{R_0}\right)^4 = \beta^4$$

40% REDUCTION

$$\beta = 60\% = 0.6 \qquad \beta^4 = 0.13 = 13\%$$

90% REDUCTION

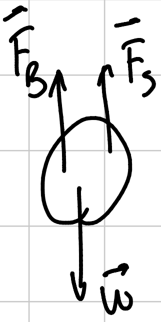
$$\beta = 10\% = 0.1 \qquad \beta^4 = 0.0001 = 0.01\%$$

STOKE'S LAW חוק סטוקס

$$F_s = 6\pi\eta R v$$

VISCOUS FORCE

דוגמה לכוח גרר ← $\propto v^1$



$$F_B = \frac{4}{3}\pi R^3 \rho_{\text{נוזל}}\, g$$

$$W = \frac{4}{3}\pi R^3 \rho_{\text{גוף}}\, g$$

TERMINAL VELOCITY

$$W = F_B + F_s \longrightarrow \frac{4}{3}\pi R^{3} g\left(\rho_{\text{גוף}} - \rho_{\text{נוזל}}\right) = 6\pi\eta R v^*$$

$$v^* = \frac{2}{9} R^2 g \frac{1}{\eta}\left(\rho_{\text{גוף}} - \rho_{\text{נוזל}}\right)$$

EXAMPLE

POISEUILLE'S EQUATION

דוגמה לאנליזה מימדית:

$$F = \eta A \frac{v}{l}$$

$$\eta = \frac{Fl}{Av}$$

$$[\eta] = \frac{M L T^{-2} L}{L^2 \, L T^{-1}} = M T^{-1} L^{-1}$$

$$[P] = \frac{[F]}{[A]} = \frac{M L T^{-2}}{L^2} = M L^{-1} T^{-2}$$

$$Q = \frac{1}{L} R^{\alpha} \Delta P \frac{1}{\eta}$$

$$[Q] = L^3 T^{-1} = L^{-1} L^{\alpha} M L^{-1} T^{-2} M^{-1} T L$$

$$L^3 T^{-1} = L^{\alpha-1} T^{-1}$$

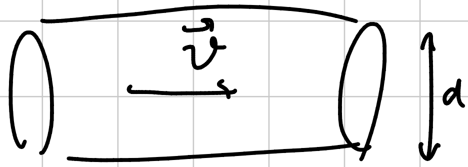
$$L^3 = L^{\alpha-1}$$

$$\alpha - 1 = 3$$

$$\alpha = 4$$

REYNOLD'S NUMBER

$$Re = \frac{\rho v d}{\eta} \qquad \frac{\text{INERTIAL}}{\text{VISCOUS}}$$



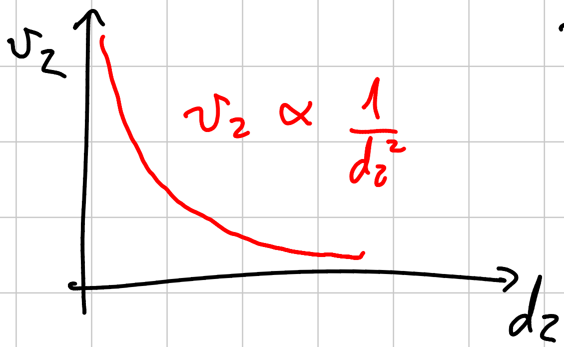
$$[Re] = \frac{M L^{-3} L T^{-1} L}{M L^{-1} T^{-1}} = 1 \qquad \text{NONDIMENSIONAL}$$

d = DIAMETER

$$A_1 v_1 = A_2 v_2$$

$$\left(\frac{d_1}{2}\right)^2 \pi v_1 = \left(\frac{d_2}{2}\right)^2 \pi v_2 \rightarrow v_2 = v_1 \left(\frac{d_1}{d_2}\right)^2$$

$$v_2 d_2 = v_1 d_1^2 \frac{1}{d_2}$$



$$Q = vA = v\left(\frac{d}{2}\right)^2 \pi \rightarrow v = Q \, \frac{4}{\pi} \frac{1}{d^2}$$

$$Re = \frac{\rho v d}{\eta} = \frac{\rho Q}{d \eta} \frac{4}{\pi}$$